Abstracts

Controllability of degenerate coupled parabolic system by one boundary control
B. Allal
Thursday, February 27th, 16.15-16.50

In collaboration with:
- M. González Burgos (IMUS Sevilla)
- A. Hajjaj and J. Salhi (FST Settat)

We study the controllability properties for one dimensional linear system of two coupled degenerate parabolic equations with one control force acting at the boundary of the space domain. We give necessary and sufficient conditions for the approximate and null controllability results. Our proofs are based on the moment method together with some properties of Bessel functions and their zeros

References

[1] P. Cannarsa, P. Martinez, and J. Vancostenoble, The cost of controlling weakly degenerate parabolic equations by boundary controls, Mat. Control Relat. Fields, 7 (2017), 171-211.

[2] H.O. Fattorini, D.L. Russell (1971) , Exact controllability theorems for linear parabolic equations in one space dimension, Arch. Ration. Mech. Anal. 43 272292.

[3] E.Fernández-Cara, M. González-Burgos, L. de Teresa (2010) Boundary controllability of parabolic coupled equations, J. Funct. Anal. 259, no. 7, 17201758.

[4] M. Gueye (2014), Exact boundary controllability of 1-D parabolic and hyperbolic degenerate equations, SIAM J. Control Optim Vol 52 , No 4, p. 2037-2054

Boundary regularity results for a fluid-structure interaction, with application to quadratic optimal control on an infinite time horizon
F. Bucci
Thursday, February 27th, 15.10-15.45

The talk will deal with a well-recognized Partial Differential Equations system which describes a fluid-structure interaction. The original mathematical model comprises a Stokes system (for the fluid) and the Lamé system of dynamic elasticity (for a solid immersed in the fluid), with the coupling occurring on an interface between the two media. A proper variational formulation of the actual nonlinear problem, and a proof of its well-posedness in the natural energy space – a highly non trivial question –, was first given by Barbu et al. in 2007. While the original linear dynamics is strongly stable, it becomes uniformly (exponentially) stable in the presence of a suitable dissipation term on the interface; cf. Avalos and Triggiani (2009). For this latter variant it makes sense to study the quadratic optimal control problem on an infinite time horizon. Suitable boundary regularity estimates that pertain to the fluid component of the system are shown to be valid, thereby ensuring solvability of the optimization problem along with uniqueness of the corresponding Algebraic Riccati Equations, in view of the theory in Acquistapace et al. (2013, 2020). The said trace regularity results, which complement the ones obtained for the undamped model (cf. Bucci and Lasiecka (2010)), follow via semigroup and interpolation methods only.

Parameter determination for energy balance models with memory
P. Cannarsa
Friday, February 28th, 13.00-13.35

In climate dynamics, more specifically in the classical Energy Balance Models introduced independently by Budyko and Sellers, one describes the evolution of temperature as the effect of the balance between the amount of energy received from the Sun and radiated from the Earth. Such models were developed to understand the past and future climate and its sensitivity to some relevant parameters on large time scales (centuries). In order to take into account the long response times that cryosphere exhibits (for instance, the expansion or retreat of huge continental ice sheets occurs with response times of thousands of years), it is interesting to let the co-albedo function depend on a memory term as in [K. Bhattacharya, M. Ghil, I. L. Vulis, *Internal variability of an energy-balance model with delayed albedo effects*, J. Atmos. Sci., Vol 39 (1982), p. 1747-1773]. The mathematical analysis of such quasilinear Energy Balance Models with Memory has been the subject of many deep works for a long time (see, e.g., [J. I. Diaz, *On the mathematical treatment of energy balance climate models*, NATO ASI Ser. Ser. I Glob. Environ. Change, 48, Springer, Berlin, 1997.]). This talk will be focussed on the results obtained in collaboration with M. Malfitana and P. Martinez in [*Parameter determination for Energy Balance Models with Memory*, Springer INdAM Series, In press (hal-01834739)], where the problem of recovering the insolation function (which is a part of the incoming solar flux) from measurements of the solution has been addressed for class of semilinear Budyko and Seller type models.

Permuting quantum eigenmodes by a quasi-adiabatic motion of a potential wall

A. Duca

Friday, February 28th, 10.10-10.45

We study the Schrödinger equation $i\partial_t\psi = -\Delta\psi + V\psi$ on $L^2((0,1),\mathbb{C})$ where $-\Delta$ is the Dirichlet Laplacian and V is a very high and localized potential wall. We aim to perform permutations of the eigenmodes and to control the solution of the equation. We consider the process where the position and the height of the potential wall change as follows. First, the potential increases from zero to a very large value, so a narrow potential wall is formed that almost splits the interval into two parts; then the wall moves to a different position, after which the height of the wall decays to zero again. We show that even though the rate of the variation of the potential's parameters can be arbitrarily slow, this process alternates adiabatic and non-adiabatic dynamics, leading to a non-trivial permutation of the eigenstates. Furthermore, we consider potentials with several narrow walls and we show how an arbitrarily slow motion of the walls can lead the system from any given state to an arbitrarily small neighborhood of any other state, thus proving the approximate controllability of the above Schrödinger equation by means of a soft, quasi-adiabatic variation of the potential.

This is a joint work with Romain Joly and Dmitry Turaev

Carleman estimates and inverse problems for transport equations

G. Floridia

Thursday, February 27th, 14.35-15.10

Let us consider the transport equation

$$\partial_t u(x,t) + (H(x)\cdot\nabla u(x,t)) + p(x)u(x,t) = 0 \quad \text{in} \quad \Omega\times(0,T),$$

where $\Omega\subset\mathbb{R}^d$ is a bounded domain. In this talk we study two inverse problems which consist of determining a vector-valued function $H(x)$ or a real-valued function $p(x)$ by initial values and data on a subboundary of Ω. In particular, in [1] we obtain the local conditional stability of Hölder type through a suitable local Carleman estimate (see also [2]).

This is a joint work with Piermarco Cannarsa (Università di Roma "Tor Vergata", Italy), Fikret Gölgeleyen (Zonguldak Bulent Ecevit University, Turkey) and Masahiro Yamamoto (The University of Tokyo, Japan).

References

[1] P. Cannarsa, G. Floridia, F. Gölgeleyen and M. Yamamoto, *Inverse coefficient problems for a transport equation by local Carleman estimate*, Inverse Problems (IOS Science), 35 no. 10 (2019).

2] P. Cannarsa, G. Floridia and M. Yamamoto, *Observability inequalities for transport equations through Carleman estimates*, Springer INdAM Series Vol. 32 (Special Issue, editors F. Alabau-Boussouira, F. Ancona, A. Porretta and C. Sinestrari), 69-87 (2019).

Memory effects and glass relaxation in integro-differential equations
P. Loreti
Friday, February 28th, 9.00-9.35

In this talk we discuss a glass relaxation model using an approximation of the stretched exponential function. Mainly, the analysis concerns wave equation with memory terms in a disk. This is a joint work with Daniela Sforza

References

[1] P. Loreti and D. Sforza, *Viscoelastic aspects of glass relaxation models* , Phys. A 526 (2019).

Mild and weak solutions of Mean Field Games problems for linear control system
C. Mendico
Friday, February 28th, 12.25-13.00

In this talk I will present recent results obtained in collaboration with P. Cannarsa about the Mean Field Games problem when the dynamic of the agents is governed by a linear controlled ODE. In particular, I will show that, by using the Lagrangian formulation, the problem has at least one solution given by the so-called Lagrangian equilibrium and that under the classical monotonicity assumption this equilibrium is unique. Now, associated with such equilibrium, call it η, we can define a mild solution of the problem given by a pair (m, V) where $m : [0, T] \mapsto \mathcal{P}(\mathbb{R}^d)$ with $m_t = e_t \sharp \eta$ and V Is the value function of the underlying optimal control problem. At this point, we prove that for a general equilibrium we have that the measure solution is $\frac{1}{2}$-H'older continuous in time and the value function is semi-concave with fractional modulus. Moreover, we proved that under a suitable additional assumption there exists an equilibrium such that the measure solution is Lipschitz continuous in time and the value function is linearly semi-concave. In conclusion, we studied the Mean Field Games system associated with such problem classically defined as a backward Hamilton-Jacobi equation coupled with a continuity equation. For this system we define the notion of weak solution, that is a pair (m, V) where $m : [0, T] \mapsto \mathcal{P}(\mathbb{R}^d)$ is a solution in the sense of distribution of the continuity equation and V is a continuous viscosity solution of the Hamilton-Jacobi equation. At last, we showed that the class of mild solutions of the Mean Field Games problem and the class of weak solutions of the Mean Field Games system coincide.

Consensus results for some Hegselmann-Krause opinion fomation models with time delay
A. Paolucci
Thursday, February 27th, 16.50-17.25

The Hegselmann-Krause model has been introduced to describe the dynamics of opinions in a population of N agents. It's natural to introduce a time delay in the model to take into account a reaction time or a time for each agent to receive information from other agents.

I will consider a Hegselmann-Krause model with non-symmetric interaction potential and time delay. By using a Lyapunov functional approach, I will prove convergence to consensus if the time delay satisfies a suitable smallness assumption.

The infinite-dimensional model, obtained by taking the mean field limit of the discrete system, is also analyzed. Using the fact that the constants appearing in the consensus estimates for the particle model are independent of the number of agents N, we can find an exponential consensus result for the transport equation.

Furthermore, I will illustrate also a modified Hegselmann-Krause model with distributed time delay, in which agents are influenced by others along an entire time-interval $[t - \tau(t), t]$.

Finally, I will show some numerical tests. These results are based on a joint work with Y.-P. Choi (Yonsei University) and C. Pignotti (Università degli Studi dell'Aquila).

Flocking estimates for Cucker-Smale models with time delay effects

C. Pignotti
Thursday, February 27th, 17.25-18.00

We describe Cucker-Smale type models with normalized weights and distributed time delay. Using appropriate Lyapunov functionals, we give sufficient conditions for the asymptotic flocking behavior. We then show that as the number of individuals N tends to infinity, the N-particle system can be well approximated by a delayed Vlasov alignment equation. We study the global existence of measure-valued solutions for the delayed Vlasov equation and analyze its large-time asymptotic behavior.

On some classes of differential operators with interior degeneracy
S. Romanelli
Thursday, February 27th, 14.00-14.35

Let us consider the operators $A_n u := (-1)^n (a u^{(n)})^{(n)}, 1 \leq n \in \mathbb{N}$, where the coeffcient a has interior degeneracy in the sense that $a \in C[0,1]$ and there exists $x_0 \in (0,1)$ such that $a(x_0) = 0$ and $a(x) > 0$ for any $x \in [0,1] \setminus \{x_0\}$. According to different types of degeneracy and boundary conditions, we show that there exist suitable domains of A_n which allow to find generation results of analytic semigroups. Connections with identification problems and controllability will be also presented.

Stochastic Proximal Gradient Methods for Nonconvex Problems in Hilbert Spaces and applications to PDE-constrained optimization with uncertainty
T. Scarinci
Friday, February 28th, 11.15-11.50

Stochastic approximation methods have long been used to solve stochastic optimization problems in finite-dimension. Their application to infinite dimensional problems is less understood, particularly for nonconvex objectives. This talk presents convergence results for the stochastic proximal gradient method applied to Hilbert spaces, motivated by optimization problems with partial differential equation (PDE) constraints with random inputs and coefficients. We study stochastic algorithms for nonconvex and nonsmooth problems, where the nonsmooth part is convex and the nonconvex part is the expectation, which is assumed to have a Lipschitz continuous gradient. We prove almost sure convergence of strong limit points of the random sequence generated by the algorithm to stationary points. We demonstrate the stochastic proximal gradient algorithm on a tracking-type functional with a sparse L1-penalty term constrained by a semilinear elliptic PDE and box constraints, where input terms and coefficients are subject to uncertainty.

Special regularity features for wave equations with memory
D. Sforza
Friday, February 28th, 9.35-10.10

The main purpose of the talk is to show how well-known regularity results available in the literature for wave equations can be extended to integrodifferential equations, which are typical examples of models in the theory of viscoelasticity. Assuming rather general conditions on the integral kernel we can define the trace of the normal derivative of the solution by the choice of suitable multipliers. Joint works with **Paola Loreti**.

Stabilization and exact controllability of parabolic equations via bilinear control
C. Urbani
Friday, February 28th, 11.50-12.25

In this talk I will first present a result of rapid stabilizability of parabolic equations of the following kind

$$\begin{cases} \dot{u}(t) + Au(t) + p(t)Bu(t) = 0; \\ u(0) = u_0; \end{cases} \tag{1}$$

to the ground state solution by means of a bilinear control p. This property can be seen as a weak version of exact controllability to a trajectory. Namely, we are able to construct a control that brings the solution of the equation arbitrary close to the ground state solution with doubly-exponential rate of convergence. Moreover, by strengthening the assumption on the operator B, we prove that it is possible to steer the solution of (1) exactly to the ground state solution in any arbitrary time $T > 0$.

Joint work with **F. Alabau, P. Cannarsa**.