UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

CONSIGLIO INTERCLASSE IN MATEMATICA

COURSE OF STUDY — TWO-YEAR MASTER OF SCIENCE PROGRAMME IN MATHEMATICS

ACADEMIC YEAR — 2023-2024

ACADEMIC SUBJECT — ADVANCED COURSE IN GEOMETRY 1

General information	
Programme year	Second
Term	First semester (September 25, 2023 – December 22, 2023)
European Credit Transfer and Accumulation System credits (ECTS)	7
SSD	MAT/03 – Geometry
Language	Italian
Mode of attendance	Not mandatory

Lecturers	
Name and surname	Maria Falcitelli
E-mail	maria.falcitelli@uniba.it
Telephone	+39 080 544 2844
Department and office	Department of Mathematics, room 9 third floor
Virtual meeting room	
Web page	https://www.dm.uniba.it/it/members/falcitelli
Office hours	Days and times have to be arranged by e-mail.

Work schedule				
	Total	Lectures	Hands-on learning	Self-study
Hours	175	56		119
ECTS credits	7	7		

Learning objectives	
	Acquiring new concepts and basic methods occurring in modern Differential Geometry, in particular in Riemannian Geometry.

Course prerequisites	
	Mathematical knowledge acquired during the first degree in Mathematics. In particular: linear algebra, tensor calculus, general Topology, classical Mathematical Analysis, projective Geometry, basic concepts occurring in Differential Geometry.

Syllabus	
Course contents	Course program Fundamental examples of smooth manifolds. The Euclidean space R^n. The sphere $S^n(r)$. The real projective space $P_n(R)$ and the antipodal map. The hyperbolic space H_n^r. The tensor algebra of a manifold. The tensor algebra on a vector space. Tensor fields of type (r,s) on a manifold: definition and properties. The tensor algebra of a manifold. Contractions. Symmetric, skew-symmetric tensors on a vector space.

	Symmetric tensor fields, differential forms on a manifold. The exterior product and the algebra of differential forms. The exterior differential. Derivations of the tensor algebra. Definition and main properties of a derivation of the tensor algebra. Examples: the derivation associated with a (1,1)-tensor field, the Lie derivative with respect to a vector field. A representation theorem of derivations. Linear connections. Definition of a linear connection. The covariant derivative of a tensor field with respect to a connection. The canonical connection on R^n. The localizability property and a representation theorem. The covariant derivative of a vector field along a curve. Parallel vector fields, geodesic curves: definition and equations. The parallel transport along a curve. The torsion and the curvature tensors of a connection. Symmetric, flat connections. Bianchi identities. Riemannian manifolds. Riemannian metrics on a manifold. The metric induced on a submanifold of a Riemannian manifold. Examples. The scalar product of two tensor fields. The musical isomorphisms. The gradient od a smooth function. The Levi-Civita connection on a Riemannian manifold and the Christoffel symbols. Examples. The parallel transport along a curve induced by the Levi-Civita connection. The distance between two points in a Riemannian manifold. Complete, geodesically complete manifolds. Conformal changes of a metric. Riemannian curvature. The Riemannian curvature tensor: definition and properties. Sectional curvatures. Manifolds with pointwise sectional curvature. The Schur lemma. Space-forms: definition and main examples. Riemannian covering spaces. Example: the n-sphere as a Riemannian covering of $P_n(R)$. Complete, connected, simply connected space-forms: a classification theorem. Ricci tensor and scalar curvature. Einstein manifolds. A characterization of Einstein manifolds in dimension 3. Riemannian submanifolds. Riemannian submanifolds of a Riemannian manifold: definition and examples. The normal bundle, normal vector fields. The Gauss and Weingarten equations. The second fundamental form, the Weingarten operators: definition and properties. The mean curvature vector. Totally geodesic, totally umbilical, minimal submanifolds. Principal curvatures. Some curvature properties of a submanifold: Gauss, Codazzi, Ricci equations. Hypersurfaces in R^{n+1}.
Reference books	T. Aubin: A course in Differential Geometry, American Mathematical Society B. Y. Chen: Geometry of submanifolds, Marcel Dekker W. Klingenberg: Riemannian Geometry, Walter de Gruyter S. Kobayashi, K. Nomizu: Foundations of Differential Geometry, Vol. I,II, Interscience Publishers G. Walschap: Metric structures in Differential Geometry, Springer.

Additional course materials	
Repository	

Expected learning outcomes	
Knowledge and understanding	Acquiring new concepts and methods of proof.
Applying knowledge and understanding	The acquired knowledge is useful in various contexts, such as in Theoretical Physics and in Mathematical Physics.
Soft skills	*Making judgements*: Ability to analyse the consistency of the argument used in a proof
	Communication skills: Students should acquire the mathematical formalism needed for comprehend textbooks and analyse advanced problems.
	Learning skills: Relating the main concepts occurring in various Mathematical and Physical disciplines. Ability in comprehend textbooks and scientific articles.

Teaching methods	
	Lessons in the lecture-room

Assessment	
Assessment methods	
Evaluation criteria	• *Knowledge and understanding*: Knowledge of the theoretical concepts and the methods of proof. • *Applying knowledge and understanding*: Ability in explaining the subject and appropriate examples. • *Making judgements*: Ability in applying the main results to problems proposed during the colloquium • *Communication skills*: Present the arguments with a correct language *Learning skills*: Ability in explaining the interrelation of the subject with basic concepts occurring in other courses.
Grading policy	The exam is passed if the mark is greater or equal to 18/30

Further information	