COURSE OF STUDY	**THREE-YEAR BACHELOR PROGRAMME IN MATHEMATICS**
ACADEMIC YEAR	**2023-2024**
ACADEMIC SUBJECT	**GEOMETRY 4**

General information	
Programme year	Second
Term	Second semester (February 26, 2024 – May 31, 2024)
European Credit Transfer and Accumulation System credits (ECTS)	8
SSD	MAT/03 – Geometry
Language	Italian
Mode of attendance	Not mandatory

Lecturers		
Name and surname	Maria Falcitelli (instructor of record)	Sara Azzali
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Department and office	Department of Mathematics room 9 third floor	Department of Mathematics room 22 second floor
Virtual meeting room		
Web page	https://www.dm.uniba.it/it/members/falcitelli	https://www.dm.uniba.it/it/members/azzali
Office hours	by appointment via email	by appointment via email

Work schedule				
	Total	Lectures	Hands-on learning (recitations)	Self-study
Hours	200	48	30	122
ECTS credits	8	6	2	

Learning objectives	
	The aim of the course is to provide basic knowledge of General Topology, including main examples of spaces and the study of topological properties.

Course prerequisites	
	Mathematical tools studied during the first year. In particular: elementary set theory, quotient spaces, continuous real functions, affine and projective spaces.

Syllabus	
Course contents	Course program Topological spaces. A topology on a set: definition and examples. Bases for a topological space. The topology generated by a base. Neighborhoods of a point, neighborhoods systems. The topology generated by a neighborhood system. Axioms of countability. Metric spaces. Definition of a metric space and examples. Open balls. The topology induced

	by a metric. Metric spaces satisfy the first axiom of countability. Equivalent metrics. The distance from a point to a set. Subsets of a topological space. The interior, the exterior and the boundary of a set. Closed sets. The closure of a set. The link between the closure, the boundary and the exterior. Examples. Dense sets and separable spaces. Continuous mappings. Definition, characterization and examples of a continuous mapping. Open mappings. Homeomorphisms: definition, characterization and examples. The group of homeomorphisms of a topological space. Topological properties. Subspaces. The topology induced on a subset. Subspaces of a topological space. Convex sets. Subspaces and continuous mappings. Product, quotient spaces. The product topology of n topologies. The canonical projections of the product space are continuous and open mappings. A characterization for continuous mappings taking values in a product space. The quotient topology on a set relative to a map and its universal property. Identifications. The equivalence relation associated with an identification. The canonical topology of a real or complex projective space. The Möbius band. Axioms of separation. Frèchet spaces. Hausdorff spaces. Examples. Limit point of a sequence of points. The uniqueness theorem for the limit point in an Hausdorff space. Regular spaces. Normal spaces. Examples. Continuous mappings and the axioms of separation. A characterization theorem for Hausdorff quotient spaces. Real projective spaces are Hausdorff . Compact spaces. Open covers of a topological space. Definition and characterization of a compact space. Closed subsets of a compact space. Compact subspaces of a metric space. Continuous mappings whose domain is a compact space. Examples. The normality property of a compact, Hausdorff space. Examples. Connected spaces. Definition and characterization of a connected (disconnetted) space. Connected subspaces. The connected subsets of the real line. Continuous mappings whose domain is a connected space. The mean value theorem. The product space of n connected spaces. Examples. The connected component of a point: definition and properties. A characterization of connected spaces involving connected components. Examples. Pathwise connected spaces. Definition of a pathwise connected space. Pathwise connected spaces are connected. Continuous mapping whose domain is pathwise connected. Locally Euclidean spaces. The main examples of pathwise connected spaces: convex subsets of the Euclidean space, connected and locally Euclidean spaces, the product of n pathwise connected spaces.
Reference books	A. Loi - Introduzione alla Topologia generale, Aracne Editrice, 2013. B.M. Manetti – Topologia, Springer-Verlag Italia, 2014. C.E. Sernesi – Geometria 2, Bollati Boringhieri, 1994. D.G. Campanella – Esercizi di topologia generale, Aracne Editrice, 1992. M. H. Mortad – Introductory Topology (Exercises and Solutions), World Scientific, 2017.
Additional course materials	

Repository	

Expected learning outcomes	
Knowledge and understanding	Fundamental concepts of general Topology and Mathematical methods useful to construct examples of topological spaces and to acquire new proof techniques.
Applying knowledge and understanding	The acquired knowledge is useful in several Mathematical disciplines, as well as in related contexts, namely Topography and Graph Theory.
Soft skills	*Making judgements*: Students must develop critical thinking in order to solve new problems, also with a theoretical approach.
	Communication skills: Acquiring appropriate language and mathematical formalism.
	Learning skills: Acquiring methods useful to link the main concepts occurring in several Mathematical disciplines and to solve new problems.

Teaching methods	
	Classroom lectures will include exercises.

Assessment	
Assessment methods	Oral exam. Firstly, the students solve an exercise, explaining the involved techniques. Then, they explain theoretical results and related examples.
Evaluation criteria	• *Knowledge and understanding*: The students reveal the knowledge of the basic tools of general Topology, explaining proof in a rigorous way. • *Applying knowledge and understanding*: The students solve exercises using appropriate methods. • *Making judgement*: The students show a critical knowledge of the treated subject. • *Communication skills*: The students are able to organize an exhaustive exposition of the subject. • *Learning skills*: The main tools of general Topology occur in several Mathematical disciplines. The students have to link the main concepts to the ones treated in other courses.
Grading policy	The exam is passed if the grade is greater or equal to 18/30

Further information	