UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

CONSIGLIO INTERCLASSE IN MATEMATICA

COURSE OF STUDY	**TWO-YEAR MASTER OF SCIENCE PROGRAMME IN MATHEMATICS**
ACADEMIC YEAR	**2023-2024**
ACADEMIC SUBJECT	**GEOMETRIC STRUCTURES ON MANIFOLDS**

General information	
Term	Second semester (February 26, 2024 – May 31, 2024)
European Credit Transfer and Accumulation System credits (ECTS)	4
SSD	MAT/03 – Geometry
Language	Italian
Mode of attendance	Not mandatory

Lecturer	
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Virtual meeting room	Microsoft Teams
Web page	https://www.dm.uniba.it/it/members/dileo
Office hours	By appointment via email

Work schedule				
	Total	Lectures	Hands-on learning	Self-study
Hours	100	32		68
ECTS credits	4	4		

Learning objectives	
	Knowledge of the most studied geometric structures on differentiable manifolds, with particular attention to Riemannian manifolds.

Course prerequisites	
	Basic knowledge in differential geometry: differentiable manifolds, tangent and cotangent spaces, tangent bundle. Tensorial algebra and tensorial calculus. Elements of Riemannian geometry.

Syllabus	
Course contents	**Complex vector spaces**. Complexification of real vector spaces and real linear maps. Complexification of the dual vector space. Complex structures on real vector spaces. Canonical complex structure on R^{2n}. C-linear maps between complex vector spaces. GL(n,C) as subgroup GL(2n,R). Positively oriented bases in a complex vector space. Complexification of a complex vector space. Vectors and forms of type (1,0) and (0,1). Decomposition of the complexified exterior algebra. **Almost complex manifolds.** Almost complex structures on differentiable manifolds. Adapted frames. Orientability of almost complex manifolds.

	Canonical almost complex structure on C^n. Vector fields and 1-forms of type (1,0) and (0,1). Decomposition of the complexified exterior bundle. The Nijenhuis tensor associated to an almost complex structure: necessary and sufficient conditions for its vanishing. **Complex manifolds.** Holomorphic functions and Cauchy-Riemann equations. Complex manifolds. Holomorphic maps between complex manifolds. Examples: complex projective space, S^2, biholomorphism between S^2 e CP^1. Canonical almost complex structure on a complex manifold. Adapted local frames and coframes. The Newlander-Nirenberg Theorem. Differential operators on complex manifolds. Characterizations of holomorphic functions. **Almost Hermitian manifolds.** Almost Hermitian manifolds. Existence of Hermitian metrics. C^n as a Hermitian manifold. Adapted local orthonormal frames. Non-degeneracy of the fundamental 2-form. The Levi-Civita connection: covariant derivatives of the almost complex structure and the fundamental 2-form. The Nijenhuis tensor of an almost Hermitian manifold. Some classes of almost Hermitian manifolds. Holomorphic sectional curvature. **Kähler manifolds.** Definition and characterization of Kähler manifolds. Kähler structures on 2-dimensional Riemannian oriented manifolds. Riemannian curvature properties for a Kähler manifold. Kähler manifolds with constant sectional curvature. Kähler manifolds with constant holomorphic sectional curvature and their classification.
Reference books	– Kobayashi S., Nomizu K.: Foundations of Differential Geometry vol.1, Wiley-Interscience, 1996. – Moroianu A., Lectures on Kähler geometry. London Mathematical Society Student Texts, 69. Cambridge University Press, Cambridge, 2007.
Additional course materials	E-learning platform
Repository	

Expected learning outcomes	
Knowledge and understanding	Acquiring the fundamental notions concerning almost complex, complex and Hermitian structures on differentiable manifolds, allowing to comprehend advanced textbooks and recent publications in currently investigated research fields.
Applying knowledge and understanding	Acquiring proof techniques in complex and Hermitian geometry, together with the knowledge of fundamental examples.
Soft skills	*Making judgements*: Ability to analyze the consistency of mathematical arguments, under the formal, logical and technical point of view. Students should become able to prove autonomously properties dealing with the program topics.
	Communication skills: Students should acquire the mathematical language and formalism necessary to the comprehension and the exposition of concepts and results concerning the studied theory.

	Learning skills: Improve learning methods acquired during previous years, through the practice in exposing results, solving problems and bibliographic search.

Teaching methods	
	Classroom lectures.

Assessment	
Assessment methods	Oral exam.
Evaluation criteria	• *Knowledge and understanding:* knowledge of the fundamental notions of complex and Hermitian geometry, together with the ability to prove their properties. • *Applied knowledge and understanding:* Ability to solve problems and illustrate the acquired notions in specific examples. • *Making judgment*: Ability to evaluate the consistency of the arguments used in a proof, and to compare alternative proofs. Capacity to ask questions and propose solutions. • *Communication skills*: Ability to expose theorems, proofs, questions, through a suitable language and mathematical formalism. • *Learning skills*: Ability to consult advanced texts and scientific articles, also written in English.
Grading policy	The final grade is out of thirty. The exam is passed when the grade is greater or equal to 18/30.

Further information	