

<b>COURSE OF STUDY</b>	<b>THREE-YEAR BACHELOR PROGRAMME IN MATHEMATICS</b>
<b>ACADEMIC YEAR</b>	<b>2023-2024</b>
<b>ACADEMIC SUBJECT</b>	<b>ELEMENTS OF ADVANCED GEOMETRY 1</b>

General information	
Programme year	Third
Term	First semester (September 25, 2023 – December 22, 2024)
European Credit Transfer and Accumulation System credits (ECTS)	7
SSD	MAT/03 – Geometry
Language	Italian
Mode of attendance	Not mandatory

Lecturer	
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Department and office	Department of Mathematics, room 5 second floor
Virtual meeting room	Microsoft Teams – elearning platform
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Office hours	By appointment via email

Work schedule				
	Total	Lectures	Hands-on learning (recitations)	Self-study
<b>Hours</b>	175	40	30	105
<b>ECTS credits</b>	7	5	2	

Learning objectives	
	Acquiring fundamental notions concerning classical differential geometry of curves and surfaces, and the geometry of differentiable manifolds.

Course prerequisites	
	Algebra and basic linear algebra. Differential calculus. Basic notions in topology.

Syllabus	
Course contents	<b>Curves.</b> Parametrized curves in $\mathbb{R}^n$ . Support, tangent vector at a point. Regular curves. Vector fields along a curve. Tangent vector field. Length of a curve. Arc length reparametrization. Differentiable curves: atlases, differentiability of transition functions, examples. Frenet frame. Existence and uniqueness of the canonical Frenet frame. Frenet equations and their behavior with respect to changes of variable. Curvature functions and their behavior with respect to isometries. Existence and uniqueness up to isometries of curves in $\mathbb{R}^n$ with assigned curvature functions. Plane curves: examples and properties. Characterization of straight lines and circular arcs. Space curves: curvature, torsion, and characterization of curves with vanishing torsion. Projections of a curve on the osculating, normal, and



rectifying planes. Examples.

**Surfaces.** Parameterized surfaces in  $\mathbb{R}^3$ . Regular surfaces and the tangent space at a point. Vector fields along a surface: tangent vector fields and normal vector fields. Gauss map. Changes of variables. Differentiable surfaces, atlases, and examples. Level surfaces. I fundamental form. Weingarten map. II fundamental form. Behavior with respect to changes of variables and isometries. Curves on a regular surface. Coordinate lines. Meusnier's theorem. Normal curvature. Principal curvatures and principal curvature directions. Umbilical points and characterization of surfaces whose points are umbilical. Gaussian curvature and mean curvature. Elliptic, hyperbolic, parabolic, and planar points. Asymptotic directions. Asymptotic lines and lines of principal curvature. Examples: parametrizations of the sphere, torus, surfaces of revolution, helicoid and ruled surfaces.

**Differentiable manifolds.** Topological manifolds: definitions, examples and first properties. Differentiable manifolds: local charts, transition functions, differentiable atlases, maximal atlas. Examples of differentiable manifolds: differentiable curves and surfaces, manifolds of dimension 0,  $\mathbb{R}^n$ , finite-dimensional real vector spaces, open submanifolds of a differentiable manifold, general linear group,  $n$ -dimensional sphere,  $n$ -dimensional real projective space, product manifolds,  $n$ -dimensional torus. Differentiable functions on manifolds. Differentiable maps between manifolds. Diffeomorphisms. Examples. Topological properties of differentiable manifolds. Partition of unity. Existence of a partition of unity subordinate to an open covering. Extensions of differentiable functions. Bump functions on manifolds.

**Tangent vectors and covectors, tangent vector fields and 1-forms.** Tangent vectors at a point. The tangent space and the coordinate basis associated to a local chart. Matrix change of coordinate bases. Tangent space to a vector space. The cotangent space. The differential of a differentiable function. The coordinate basis of the cotangent space associated to a local chart. The differential of a differentiable map between manifolds: definition and properties. Tangent space of an open submanifold. Tangent space of a product manifold. Matrix associated to the differential of a map with respect to coordinate bases. Immersions, imbeddings and submersions: definitions and examples. Differentiable curves and tangent vectors. Tangent bundle, cotangent bundle, and their sections. Differentiable vector fields. Extensions of vector fields and tangent vectors. One-to-one correspondence between differentiable sections of the tangent bundle and derivations of the algebra of differentiable functions. The Lie algebra of differentiable vector fields. Differentiable 1-forms. One-to-one correspondence between differentiable sections of the cotangent bundle and linear maps on the module of differentiable vector fields.

**Tensor fields and differential forms.** Elements of tensor algebra on a real vector space. Tensors of type  $(r,s)$  on a vector space. Symmetric and alternating tensors. Wedge product. Basis and dimension of the space of  $k$ -forms on a vector space. The Grassmann algebra. Tensor fields of type  $(r,s)$  on a differentiable manifold. Differentiable tensor fields and their local expression in a chart. Symmetric and alternating tensor fields. Grassmann algebra of differential forms. Orientable manifolds. Exterior differentiation of



	a differential form.
Reference books	<ul style="list-style-type: none"> <li>• Wilhelm Klingenberg, <i>A course in differential geometry</i>. Graduate Texts in Mathematics, 51. Springer-Verlag, New York-Heidelberg, 1978.</li> <li>• Manfredo P. do Carmo, <i>Differential geometry of curves and surfaces</i>. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1976.</li> <li>• John M. Lee, <i>Introduction to smooth manifolds</i>. Graduate Texts in Mathematics, 218. Springer-Verlag, New York, 2003.</li> <li>• Marco Abate, Francesca Tovena, <i>Geometria differenziale</i>. Unitext, 54. Springer, Milan, 2011.</li> <li>• E. Sernesi, <i>Geometria 2</i>. Bollati Boringhieri, 2019.</li> </ul>
Additional course materials	
Repository	Lecture notes on the e-learning platform.

Expected learning outcomes	
Knowledge and understanding	Acquiring fundamental concepts and results concerning the classical theory of differentiable curves and surfaces, and the geometry of differentiable manifolds.
Applying knowledge and understanding	Acquiring basic calculus techniques for differentiable curves, surfaces and manifolds. Description of remarkable examples. Acquiring basic proof techniques in differential geometry.
Soft skills	<i>Making judgements</i> : Ability to evaluate the consistency of arguments, both from a logical and a formal point of view. Ability to solve problems, even of theoretical nature.
	<i>Communication skills</i> : Acquiring a correct mathematical language in order to understand and present the results, and discuss related problems.
	<i>Learning skills</i> : Acquiring an adequate learning method, supported by the consultation of textbooks, and by the research of sources for further study.

Teaching methods	
	Lectures and exercises in presence.

Assessment	
Assessment methods	Oral exam consisting in the exposition and discussion of definitions, statements and proofs of results related to the developed programme. The exam also includes the discussion of examples and the solution of exercises.
Evaluation criteria	<ul style="list-style-type: none"> <li>• <i>Knowledge and understanding</i>: completeness and correctness in the discussion of results, examples, exercises.</li> <li>• <i>Applying knowledge and understanding</i>: ability in proving the results of the theory, and in solving problems.</li> <li>• <i>Making judgement</i>: ability in arguing in a correct way.</li> <li>• <i>Communication skills</i>: ability in using a correct language and formalism in the exposition of</li> </ul>



	results; dialogic ability. • <i>Learning skills:</i> ability to choose proper techniques and procedures in proving results and solving problem.
Grading policy	The final evaluation of the exam is expressed out of thirty. The exam is passed with a minimum grade of 18/30.

Further information	
	Attendance is strongly recommended.