General information		Academic year 2022-2023
Academic subject	**Elements of Advanced Geometry 1**	
Degree programme	Mathematics	
Programme year	Third	
Term	First semester (September 26, 2022 – December 22, 2022)	
European Credit Transfer and Accumulation System credits (ECTS)		7
Language	Italian	
Attendance	Not compulsory	

Lecturer	
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Syllabus	
Learning objectives	Acquiring fundamental notions concerning classical differential geometry of curves and surfaces, and the geometry of differentiable manifolds.
Course prerequisites	Algebra and basic linear algebra. Differential calculus. Basic notions in topology.
Course contents	**Parameterized curves.** Curves in R^n. Parametrizations, support, tangent vector at a point, length of a curve. Vector fields along a curve. Tangent vector field. Regular curves. Frenet frame. Existence and uniqueness of the canonical Frenet frame. Frenet equations and their behavior with respect to isometries and changes of variable. Curvature functions. Existence of curves in R^n with assigned curvature functions. Plane curves: examples and properties. Characterization of straight lines and circular arcs. Space curves: curvature, torsion, and characterization of curves with vanishing torsion. Projections of a curve on the osculating, normal, and rectifying planes. Examples. **Parameterized surfaces.** Parameterized surfaces in R^3. Regular surfaces and the and tangent space at a point. Vector fields along a surface: tangent vector fields and normal vector fields. I fundamental form: behavior with respect to isometries and changes of variables. Weingarten map. II fundamental form. Curves on a regular surface. Coordinate lines. Meusnier's theorem. Normal curvature. Principal curvatures and principal curvature directions. Gaussian curvature and mean curvature. Elliptic, hyperbolic, parabolic, and planar points. Umbilical points. Plane surfaces and spherical surfaces. Asymptotic directions. Asymptotic lines and lines of principal curvature. Examples: parametrizations of the sphere, torus, surfaces of revolution, helicoid and ruled surfaces. **Differentiable manifolds.** Differentiable curves and surfaces: atlases, differentiability of transition functions, examples. Topological manifolds: definitions, examples and first properties. Differentiable manifolds: local charts, transition functions, differentiable atlases, maximal atlas. Examples of differentiable manifolds: manifolds of dimension 0, R^n, finite-dimensional

	real vector spaces, open submanifolds of a differentiable manifold, general linear group, the graph of a continuous function, n-dimensional sphere, n-dimensional real projective space, product manifolds, n-dimensional torus. Differentiable functions on manifolds. Differentiable maps between manifolds. Diffeomorphisms. Examples. Topological properties of differentiable manifolds. Partition of unity. Existence of a partition of unity subordinate to an open covering. Extensions of differentiable functions. **Tangent vectors and covectors, tangent vector fields and 1-forms.** Tangent vectors at a point. The tangent space and the coordinate basis associated to a local chart. The tangent space as the space of vectors tangent to curves. The cotangent space. The differential of a differentiable function. The coordinate basis of the cotangent space associated to a local chart. Tangent bundle, cotangent bundle, and their sections. Differentiable vector fields: one-to-one correspondence between differentiable sections of the tangent bundle and derivations of the algebra of differentiable functions. The Lie algebra of differentiable vector fields. Extensions of vector fields and tangent vectors. Differentiable 1-forms: one-to-one correspondence between differentiable sections of the cotangent bundle and linear maps on the module of differentiable vector fields. The differential of a differentiable map between manifolds: definition and properties. Matrix associated to the differential of a map with respect to coordinate bases. Immersions, imbeddings and submersions: definitions and examples. **Tensor fields and differential forms.** Elements of tensor algebra on a real vector space. Tensors of type (r,s) on a vector space. Symmetric and alternating tensors. Wedge product. Basis and dimension of the space of r-forms on a vector space. The Grassmann algebra. Tensor fields of type (r,s) on a differentiable manifold. Differentiable tensor fields and their local expression in a chart. Symmetric and alternating tensor fields. Grassmann algebra of differential forms. Orientable manifolds. Exterior differentiation of a differential form.
Reference books	• Wilhelm Klingenberg, *A course in differential geometry*. Graduate Texts in Mathematics, 51. Springer-Verlag, New York-Heidelberg, 1978. • Manfredo P. do Carmo*, Differential geometry of curves and surfaces*. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1976. • John M. Lee, *Introduction to smooth manifolds*. Graduate Texts in Mathematics, 218. Springer-Verlag, New York, 2003. • Marco Abate, Francesca Tovena, *Geometria differenziale.* Unitext, 54. Springer, Milan, 2011.
Additional course materials	

Work schedule				
	Total	Lectures	Hands-on learning (exercises)	Self-study
Hours	175	40	30	105
ECTS credits	7	5	2	

Teaching methods	
	Classroom lectures and exercises.

Expected learning outcomes	
Knowledge and understanding	Acquiring fundamental concepts and results concerning the classical theory of differentiable curves and surfaces, and the geometry of differentiable manifolds.
Applying knowledge and understanding	Acquiring basic calculus techniques for differentiable curves, surfaces and manifolds. Description of remarkable examples. Acquiring basic proof techniques in differential geometry.
Making judgements	Ability to evaluate the consistency of arguments, both from a logical and a formal point of view. Ability to solve problems, even of theoretical nature.
Communication skills	Acquiring a correct mathematical language in order to understand and present the results, and discuss related problems.
Learning skills	Acquiring an adequate learning method, supported by the consultation of textbooks, and by the research of sources for further study.

Assessment and feedback	
Assessment methods	Oral exam consisting in the exposition and discussion of definitions, statements and proofs of results related to the developed programme. The exam also includes the discussion of examples and the solution of exercises.
Evaluation criteria	• *Knowledge and understanding*: completeness and correctness in the discussion of results, examples, exercises. • *Applying knowledge and understanding*: ability in prooving the results of the theory, and in solving problems. • *Making judgements*: ability in arguing in a correct way. • *Communication skills*: ability in using a correct language and formalism in the exposition of results; dialogic ability. • *Learning skills*: ability to choose proper techniques and procedures in prooving results and solving problem.
Grading policy	The final evaluation of the exam is expressed out of thirty. The exam is passed with a minimum grade of 18/30.

Additional information	