COURSE OF STUDY	**THREE-YEAR BACHELOR PROGRAMME IN MATHEMATICS**
ACADEMIC YEAR	**2023-2024**
ACADEMIC SUBJECT	**GEOMETRY 2**

General information	
Programme year	First
Term	Second semester (February 26, 2024 – May 31, 2024)
European Credit Transfer and Accumulation System credits (ECTS)	8
SSD	MAT/03 – Geometry
Language	Italian
Mode of attendance	Not mandatory

Lecturers		
Name and surname	Giulia Dileo (instructor of record)	Antonio Lotta
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Department and office	Department of Mathematics room 5 second floor	Department of Mathematics room 2 third floor
Virtual meeting room	Microsoft Teams	Microsoft Teams
Web page	https://www.dm.uniba.it/it/members/dileo	https://www.dm.uniba.it/it/members/lotta
Office hours	By appointment via email	By appointment via email

Work schedule				
	Total	Lectures	Hands-on learning (recitations)	Self-study
Hours	200	48	30	122
ECTS credits	8	6	2	

Learning objectives	
	Acquiring fundamental notions in affine and Euclidean geometry.

Course prerequisites	
	Basic knowledge of linear algebra: matrix calculus, linear systems, vector spaces, linear maps and bilinear forms.

Syllabus	
Course contents	**Euclidean vector spaces.** Review on symmetric bilinear forms and Sylvester's theorem. Dot (scalar) products on real vector spaces. Standard scalr product on R^n. Orthogonality with respect to a dot product. Orthogonal projection onto a subspace. Principal minors and Sylvester's criterion for establishing whether a symmetric bilinear form is positive definite. Norm of a vector. Cauchy-Schwarz inequality. Triangular inequality. Angle between two vectors. Orthogonal and orthonormal sequences of vectors. Orthonormal bases. Gram-Schmidt orthogonalization procedure and orthonormalization.

	Linear isometries: definition and characterizations. Orthogonal matrices. The group O(V) of orthogonal transformations of a Euclidean vector space V and the group O(n) of orthogonal matrices of order n. Transition matrices between orthonormal bases are orthogonal. *Symmetric (self-adjoint) endomorphisms* and related characterization. Eigenvalues of a real symmetric matrix. Orthogonality of eigenvectors of a symmetric endomorphism related to distinct eigenvalues. The Spectral Theorem. Existence of orthonormal diagonalizing bases for a symmetric bilinear form defined on a Euclidean vector space. Criterion for a symmetric bilinear form to be positive definite. Orientations of a real vector space. The cross product operation. Relationship between cross products and orientations of a three-dimensional Euclidean vector space. **Affine spaces.** Geometric vectors. Affine spaces associated to vector spaces: definition, elementary properties and first examples. Vector spaces as affine spaces. Affine frames and coordinate systems. Change of affine frames. Orientation of a real affine space. Midpoint of two points. **Affine subspaces.** Affine subspaces and their direction. Affine space structure induced on an affine subspace. Affine subspace spanned by a finite number of points: definition and characterizations. Affinely independent points. Collinear and coplanar points. Intersection of affine subspaces. Parallel subspaces. The affine subspace spanned by two subspaces. Affine Grassmann identity. Coplanar lines. Parametric and cartesian equations of an affine subspace. *Affine geometry in dimension 2*. Parametric equations and cartesian equation of a line. Coordinate axes. Parallel lines and intersection of lines. Sheaves of lines. *Affine geometry in dimension 3*. Parametric and cartesian equations of a plane. Parallel planes. Intersection of planes. Parametric and cartesian equations of a line. Coordinate axes and planes. Parallel lines. Parallelism between a line and a plane. Coplanar lines. Sheaves of planes. **Euclidean spaces.** Euclidean space associated to a Euclidean vector space. Cartesian frames and cartesian coordinates. Change of cartesian frames. Distance between two points. Euclidean affine subspaces. Angles between two lines. Orthogonal lines. Orthogonal subspaces. Orthogonal projection of a point onto a subspace. Distance of a point to a subspace. Formula for computing the distance of a point to a hyperplane. Distance between two affine subspaces. Hyperspheres. Reciprocal positions between a hypersphere and an affine subspace. The tangent hyperplane to a hypersphere at a given point. *Euclidean geometry in dimension 2*. Angles between lines. Orthogonal lines. Angles between a line and coordinate axes. Angular coefficient of a line.

	Distance of a point to a line. Circles. *Euclidean geometry in dimension 3.* Angles between lines. Orthogonal lines. Angles between a line and coordinate axes. Orthogonality between a line and a plane. Orthogonal planes. Distance of a point to a plane. Distance of a point to a line. Minimum distance between lines. Spheres and circles. **Affine maps and affine transformations.** Affine maps: definition and first properties. Affine transformations. The affine group. Existence and uniqueness of affine transformations. Equations of an affine transformation. Translations: definition, characterization and equations. Fixed points of an affine transformation. Homotheties and symmetries. **Isometries.** Isometries of a Euclidean space: definition and characterization. Isometries and angles between lines. Existence and uniqueness of isometries. Equations of an isometry. Direct isometries and opposite isometries. Rotations and reflections. Reflections with respect to a hyperplane. Isometries of the Euclidean plane: translations, rotations, reflections, glide reflections. Chasles' Theorem. Rotations in the 3-dimensional Euclidean space. Hints on the classification of isometries in the 3-dimensional Euclidean space. **Affine and Euclidean conics.** Conics in the real or complex affine plane. Equation of a conic. Affine and Euclidean classification of conics, and their canonical forms.
Reference books	- E. Abbena, A.M. Fino, G.M. Gianella, *Algebra lineare e geometria analitica*, Aracne. - S. Abeasis, *Algebra lineare e Geometria*, Zanichelli. - M. Audin, *Geometry*, Universitext, Springer. - M. Berger, *Geometry I*, Universitext, Springer. - G. Campanella, *Affinità, isometrie, proiettività*, Aracne. - E. Sernesi, *Geometria 1*, Bollati Boringhieri.
Additional course materials	Exercise sheets and notes on parts of the program are made available on the e-learning platform. https://elearning-mat.hosting.uniba.it/my/
Repository	

Expected learning outcomes	
Knowledge and understanding	Acquiring fundamental concepts in affine and Euclidean geometry. Acquiring basic mathematical proof techniques.
Applying knowledge and understanding	Students should become able to use the acquired theoretical knowledge in solving problems.
Soft skills	*Making judgements*: Ability to analyze the consistency of the logical arguments used in a proof. Problem solving skills should be supported by the capacity in evaluating the consistency of the found solution with the theoretical knowledge.
	Communication skills: Students should acquire the mathematical language and formalism necessary to read and comprehend textbooks, to explain the acquired knowledge, and to describe, analyze and solve problems.

	Learning skills: Acquiring suitable learning methods necessary to read and understand texbooks dealing with the program topics.

Teaching methods	
	Lectures and exercises in presence.

Assessment	
Assessment methods	The exam consists in a written test and an oral examination. The written exam consists in solving exercises, the oral exam in the exposition of definitions, statements and proofs, and the discussion of specific examples.
Evaluation criteria	● *Knowledge and understanding*: knowledge of the fundamental concepts in affine and Euclidean geometry, together with the capacity to state and prove related properties; capacity to show the acquired notions in specific examples. ● *Applying knowledge and understanding*: knowledge of how to use the acquired theoretical notions in solving exercises of affine and Euclidean geometry, including: scalar products, orthonormal bases, orthogonal complement, symmetric endomorphisms; parametric and cartesian equations of affine and Euclidean subspaces, and their geometric properties (parallelism, intersection, coplanar lines, orthogonality, angles, distances); equations of affine transformations and isometries, their properties and classification problems; equations of circles and spheres; affine and Euclidean classification of conics. ● *Making judgement*: capacity in evaluating the consistency of the logical arguments used in a proof. Problem solving skills, coherently with the acquired theoretical knowledge. ● *Communication skills*: capacity in the exposition of definitions, statements and proofs, and in presenting solutions of exercises in suitable mathematical language and formalism. ● *Learning skills*: capacity in consulting textbooks, in finding logical links and solving exercises.
Grading policy	The evaluation of the written test is based on: consistency of procedures, correctness of results, methodological rigor, correctness of mathematical formalism. The written test is given a score out of thirty. The written test is passed if the score is greater than or equal to 18/30. Passing the written test is a necessary requirement to access the oral exam. The evaluation of the oral exam is based on: completeness of the preparation, degree of detail of the program, correctness and clarity in the presentation of definitions, statements, proofs, and examples, language and mathematical formalism. The written test and the oral exam contribute to the overall evaluation of the exam. The final grade is out of thirty. The exam is passed when the mark is greater than or equal to 18/30.

Further information	
	The attendance of lectures and exercises is strongly recommended.