UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

CONSIGLIO INTERCLASSE IN MATEMATICA

General information	Academic year 2022-2023
Academic subject	**Elements of Advanced Analysis 1**
Degree programme	Mathematics
Programme year	First
Term	First semester (September 19, 2022 – December 22, 2022)
European Credit Transfer and Accumulation System credits (ECTS)	Es.: 7
Language	Italian
Attendance	Not compulsory

Lecturer	
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Syllabus	
Learning objectives	Real Analysis 1. Measure and abstract integration theory: σ–algebras, measurable sets and functions – elementary properties of the measure – integration of positive functions and complex–valued functions – sequences of integrals: Monotone Convergence Theorem, Fatou Lemma, Dominated Convergence Theorem – series of integrals – completion of a measure – Severini–Egoroff theorem – Vitali's convergence theorem. 2. Lebesgue measure in R^N : simple sets, Lebesgue outer and inner measure – Lebesgue measurable sets – existence of non–Lebesgue measurable sets in R^N – positive translation–invariant Borel measures – Lebesgue measure and linear transformations: the geometric meaning of the determinant. 3. L^p spaces: Jensen, Hölder and Minkowsky inequalities – completeness of L^p spaces – continuity properties of measurable functions in R^N: Lusin's theorem – density of $C_c(R^N)$ into $L^p(R^N)$ – density of $C_c(R^N)$ into $C_0(R^N)$. Separability of $L^p(R^N)$. 4. Elementary theory of Hilbert spaces: definition, Schwarz inequality, triangle inequality – existence of the element of smallest norm for closed convex sets – orthogonal projections – Riesz representation theorem in Hilbert spaces – the best approximation theorem – orthonormal sets, characterization of maximal orthonormal sets, existence of maximal orthonormal set – Bessel and Parseval identities, the isomorphism between H e $l^2(A)$ – the space $L^2(T)$ and the Fourier series – the spaces $H(T^N)$ e $H^s(T^N)$ and the embedding theorems into C(T) e $C(T^N)$ - Applications to differential equations and to the plane isoperimetric inequality. Complex Analysis 5. Introduction to holomorphic function theory: complex differentiability: properties, geometric meaning – olomorphy and differentiability – Cauchy–Riemann equations and corollaries – some elementary holomorphic functions: complex exponential, complex trigonometric functions, multivalued functions and selections, complex logarithm, complex power – curves, paths, contours – a summary about differential

	forms – omothopy – simply connected sets – closed and exact differential forms – path integral – primitives of complex functions – holomorphic functions and differential forms – carachterization of the existence of primitives of complex functions – complex power series: convergence radius, uniform convergence, Cauchy–Hadamard theorem – Abel–Dirichlet test – Abel's theorem – Cauchy product – analytic functions – analiticity of the Cauchy integral. 6. Cauchy Theorem and analiticity of holomorphic functions: winding number theorem – Goursat theorem – existence of local primitives – Cauchy formula – analyticity of holomorphic functions – Morera's theorem – Cauchy formula for derivatives – Cauchy estimates for derivatives – fundamental theorem of algebra – Liouville's theorem for bounded holomorphic functions and generalizations – Morera–Weierstrass theorem – calculus of integrals. 7. Zeros of holomorphic functions and properties of harmonic functions: theorem about the zeros of holomorphic functions and corollaries – uniqueness of analytic continuation – real analytic functions – relationship between holomorphic and harmonic functions – mean value property – Pizzetti's formula – characterization of harmonic functions by means of their mean value – Liouville's theorem for positive harmonic functions and generalizations – maximum principle for harmonic functions – Mean value theorem for holomorphic functions – maximum modulus principle, minimum modulus principle. 8. Residue Theorem and applications: isolated singularities – Laurent series – theorem about Laurent series developability – classification of isolated singularities and characterizations – Picard's theorem (only statement) – residues – calculus of the residue at a pole – residues theorem – Cauchy's theorem (general case) – Jordan's lemma – applications to integral calculus, series, differences equations – meromorphic functions – logarithmic index theorem – Rouché theorem and corollaries – open mapping theorem for holomorphic functions – inverse function theorem for holomorphic functions.
Course prerequisites	Acquiring language and techniques of modern analysis, especially fin measure theory, Hilbert space elementary theory, L^p spaces and analysis of complex variable
Course contents	Mathematical knowledge which usually is acquired during the first two years of a degree of L–35 class. Especially: classical analysis of one and several variables, general topology, linear algebra
Reference books	W. RUDIN, *Real and Complex Analysis*, McGraw–Hill Book Company Lebesgue measure in RN : N. FUSCO, P. MARCELLINI & C. SBORDONE, *Analisi Matematica due*, Liguori Complex analysis: G. GILARDI, *Analisi 3*, Ed. Mc Graw–Hill; S. LANG, *Complex Analysis*, Springer–Verlag
Additional course materials	Notes

Work schedule				
	Total	Lectures	Hands-on learning (recitations/laboratories /seminars/other)	Self-study
Hours	175	70	0	105
ECTS credits	7	7	0	

Teaching methods	
	Classroom lectures which include exercises whose purpose is to make the student acquire the ability to apply theoretical concepts. Due to the ongoing health emergency, teaching will take place according to the Academic Senate's resolutions

Expected learning outcomes	
Knowledge and understanding	◦ Acquiring fundamental tools of moder analysis and complex analysis ◦ Acquiring proof strategies
Applying knowledge and understanding	The acquired theoretical knowledge is useful in large part of modern mathematics and its applications
Making judgements	- Evaluate the consistency of the logic reasoning in a proof - Find the correct mathematical tools to face complex analysis problems - Find the most elegant, short, formally correct and complete strategies to solve exercises
Communication skills	- Use in correct and clear way the advanced mathematical formal language - to present the acquired knowledge, describe and solve problems - Explain in a clear way the solutions obtained for the proposed problems
Learning skills	Acquire an adequate study methodology, with bibliographic investigation, and ability to autonomously go deep in the topics, solve exercise and questions proposed

Assessment and feedback	
Assessment methods	Oral exams on Theorems and their proofs, definition, exercises.
Evaluation criteria	• *Knowledge and understanding*: - Correct and complete discussion - Appropriate use of the mathematical tools in the proofs - Correct answer to the questions during the oral test • *Applying knowledge and understanding*: Correct tools used to completely solve the problems • *Making judgements*: Best tools to solve the question or problem proposed • *Communication skills*: - Formally correct mathematical language - Clarity of the exposition and of the answers • *Learning skills*: Correct solutions to the proposed exercises
Grading policy	The exam is sufficient with 18/30 mark

Additional information	