UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

CONSIGLIO INTERCLASSE IN MATEMATICA

COURSE OF STUDY **TWO-YEAR MASTER OF SCIENCE PROGRAMME IN MATHEMATICS**

ACADEMIC YEAR **2023-2024**

ACADEMIC SUBJECT **ELEMENTS OF ADVANCED ANALYSIS**

General information	
Programme year	First
Term	Second semester (February 26, 2024 – May 31, 2024)
European Credit Transfer and Accumulation System credits (ECTS)	7
SSD	MAT/05 – Mathematical Analysis
Language	Italian
Mode of attendance	Not mandatory (strongly recommended)

Lecturer	
Name and surname	Marcello D'Abbicco
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Department and office	Department of Mathematics, room 36 second floor
Virtual meeting room	marcello.dabbicco@uniba.it
Web page	https://www.dm.uniba.it/it/members/dabbicco
Office hours	On appointment, by email. Meeting will be in the lecturer's office.

Work schedule				
	Total	Lectures	Hands-on learning (recitations)	Self-study
Hours	175	40	30	105
ECTS credits	7	5	2	

Learning objectives	
	Acquiring advanced tools of modern analysis: continuous linear functionals in normed and locally convex spaces; test functions and distributions; convolution and Fourier and Laplace transforms in functional and distributional spaces, introduction to the Sobolev spaces, fundamental solution to evolution partial differential equations.

Course prerequisites	
	Classical real analysis of one and several variables, general topology, linear algebra, Lebesgue measure and integration theory, complex analysis in one variable.

Syllabus	
Course contents	**Locally convex spaces and distributions.** Topological groups and topological vector spaces (TVS); seminormed and normed spaces; TVS topology from a family of seminorms; Banach and Fréchet spaces; examples: C(Ω), *E*(Ω), D_K, spazio di Schwartz *S*(R^n); construction of seminorms on locally convex TVS, construction and properties of the topology of *D*(Ω). Continuous linear operators on TVS, norm and completeness of the space of

	bonunded linear functionals on normed spaces; dual space of *E*(Ω), D_K, *S*(**R**^{n}); continuous linear operators on *D*(Ω); Hahn-Banach theorem in locally convex TVS and in normed spaces; bidual and reflexive normed spaces; distributions, distributional derivative; P.V. 1/x; distributions' order; multiplication of functions and distributions; distributional convergence. Exercises. **Convolution and compactly supported distributions.** Halmos theorem and construction of the product measure; Fubini and Tonelli theorems and counterexamples; applications. Riesz-Thorin interpolation theorem (without proof); Young theorem on convolution; convolutions' support; approximations of identity; and convolution; mollifiers; fundamental lemma of calculus of variations. C∞ Urysohn and unit partition; support of a distribution; characterization of compactly supported distributions; local representation theorem of distributions; convolution of functions and distributions and between distributions. Exercises. **Fourier and Laplace transforms.** Fourier transform in L^1 and properties; Gauss-Weierstrass, Fejér and Abel-Poisson kernels, inversion theorem in L^1 and in *S*; Plancherel theorem; Hausdorff-Young inequality. Laplace transform and properties; application to Cauchy problems for ordinary differential equations; initial boundary value problem for the wave equation on the half-line; Riemann-Fourier formula; initial boundary value problem for the heat equation on the half-line; Laplace transform of distributions. Exercises. **Tempered distributions, Sobolev spaces and evolution equations.** Tempered distributions and Fourier transform; Fourier transform of compactly supported distributions; Paley-Wiener theorem; Hilbert transform; Fourier transform of radial functions. Exercises. Completeness, separability and reflexivity of Sobolev spaces; Bessel potential, H^s spaces and Sobolev embeddings; $W_0^{m,p}$ (Ω) spaces; density of test functions in $W^{m,p}$ (R^n); $W^{-m,p'}$(Ω) as dual of $W_0^{m,p}$ (Ω); representation theorem of $W^{-1,p'}$(Ω). Heat equation: Duhamel's principle, fundamental solution, decay estimates; wave equation: fundamental solution, finite speed of propagation, H^s estimates, energy conservation; Klein-Gordon equation and H^s estimates; Schrödinger equation and H^s conservation; plate equation.
Reference books	W. RUDIN, Analisi funzionale, McGraw-Hill W. RUDIN, Analisi reale e complessa, Boringhieri G. GILARDI, Analisi 3, Mc Graw-Hill F. TOMARELLI, Mathematical Analysis Tools for Engineering, Esculapio
Additional course materials	
Repository	Lecture notes are available on the webpage of the lecturer.

Expected learning outcomes	
Knowledge and understanding	Acquiring fundamental tools of modern analysis and related proof strategies: normed and locally convex spaces, theory of distributions, convolution product, Fourier and Laplace transforms, fundamental solution to partial differential equations.
Applying knowledge and understanding	Ability to work with convolution, Fourier transform, and Laplace transform in functional and distributional spaces.

	Ability to employ the acquired instruments to solve and study the properties of the solution to initial value problems and initial value boundary problems for evolution partial differential equations.
Soft skills	*Making judgements*: The ability to evaluate the consistency of the logic reasoning in a proof is developed discussing the theorems' proofs in aula. The ability to find the correct mathematical tools to face complex analysis problems and the most elegant, short, formally correct and complete strategies to solve exercises, is developed solving and discussing in the lecture room the assigned exercises.
	Communication skills: The ability to use in correct and clear way the advanced mathematical formal language to present the acquired knowledge, describe and solve problems, is developed during the lectures and exercises in the lecture room. The ability to explain in a clear way the solutions obtained for the proposed problems is developed when those problems are presented to the other students in the lecture room.
	Learning skills: Acquire an adequate study methodology, with bibliographic investigation, and ability to autonomously go deep in the topics, solve exercise and questions proposed.

Teaching methods	
	Lectures, supported by slides and lecture notes, that are available in advance on the lecturer's webpage. During the lectures, exercises will be proposed, for which teamwork is allowed. Those exercises will be presented and discussed in the subsequent lectures.

Assessment	
Assessment methods	Oral test. Optionally, one to three intermediate tests are available during the course. The final assessment is based on a weighted average of the intermediate tests.
Evaluation criteria	• *Knowledge and understanding: correct and complete discussion, appropriate use of the mathematical tools in the proofs, correct answer to the questions during the oral test* • *Applying knowledge and understanding*: correct tools used to completely solve the problems • *Making judgements*: best tools to solve the question or problem proposed • *Communication skills*: formally correct mathematical language, clarity of the exposition and of the answers • *Learning skills*: correct solutions to the proposed exercises
Grading policy	The exam is sufficient with 18/30 mark.

Further information	