General information		Academic year 2022-2023
Academic subject	**Functional Analysis**	
Degree programme	Mathematics	
Programme year	Third	
Term	Second semester (February 27, 2023 – May 26, 2023)	
European Credit Transfer and Accumulation System credits (ECTS)	7	
Language	Italian	
Attendance	Strongly recommended.	

Lecturers		
Name and surname	Marcello D'Abbicco (instructor of record)	Giusi Vaira
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Web page	https://www.dm.uniba.it/members/dabbicco	https://www.dm.uniba.it/members/vaira
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Syllabus	
Learning objectives	Acquiring language and basic tools concerning functional spaces, representation theorems, operator theory, with applications to some classes of partial differential equations.
Course prerequisites	Mathematical knowledge which usually is acquired during the first two years of a degree of L 35 class. Especially: classical analysis of one and several variables, normed spaces, general topology, linear algebra.

Course contents	**1. NORMED SPACES** Normed spaces, convex sets, examples. Sobolev spaces. Norm of bounded linear operators. Finite-dimensional normed spaces. Riesz lemma and Riesz theorem on the compactness of the unit ball. Space of bounded linear operators: completeness and characterization of continuity. **2. HAHN-BANACH THEOREMS** Analytical and geometrical forms of Hahn-Banach theorem. Dual space and duality map. Bidual space and reflexive spaces. Closed subspaces of reflexive spaces. Reflexivity and separability of the dual. **3. WEAK AND WEAK* TOPOLOGIES** Initial and weak topologies. Weak convergence. Weak closure, and convexity. Weak* topology and convergence. Banach-Alaoglu-Bourbaki Theorem. Helly Lemma and Goldstine lemma. Kakutani theorem. Separability and metrizability of the unit ball in the dual w.r.t the weak* topology. Separability of the dual and metrizability of the unit ball w.r.t the weak topology (with no proof). Milman-Pettis theorem. **4. L^p SPACES** Clarkson inequality and uniform convexity of L^p spaces (the proof is only given for p in [2, ∞)). Reflexivity of L^p spaces. Properties of L^1 and L^∞, Dual space of c_0. Schur property for ℓ^1. **5. CONTINUOUS AND LINEAR OPERATORS** Linear and bounded operators and properties. Linear operators defined on normed spaces of finite dimension. The norm of an operator. The set of linear and continuous operators on Banach spaces. Neumann series. Banach-Steinhaus Theorem (the uniform boundedness principle). The effects of Banach- Steinhaus Theorem. The Open Mapping Theorem and the Closed Graph Theorem. Unbounded linear operator. Closed operators. Adjoint operators and properties. A characterization of operators with closed range. Operators with finite rank. Representation theorem and properties. The approximation of an operator and properties. The resolvent set, spectrum and point spectrum of an operator. Properties of the spectrum of a continuous and linear operator. **6. COMPACT OPERATORS** Compact operators and properties. The approximation problem. Schauder Theorem. The spectrum of a compact operator. Fredholm operators. Fredholm Alternative Theorem. Compact embedding Theorems in Sobolev Spaces. Completely continuous operators and compact operators. Applications.

	7. OPERATORS IN HILBERT SPACES Orthonormal basis in separable Hilbert spaces. Hilbert – Schmidt operators and their representation. Hilbert-Schmidt operators as compact operators. Bounded and self-adjoint operators, monotone operators, idempotent operators and normal operators. Characterization of a self-adjoint and idempotent operators. Characterization of a normal operator. Unbounded symmetric operators, self-adjoint and maximal monotone operators. Properties of a self-adjoint of the spectrum of a monotone and self-adjoint operator. Hilbert basis of eigenvectors of compact and self-adjoint operators. Resolvent operator and Yosida approximation. Solution of evolution problems. Cauchy, Lipschtz, Picard Theorem. Hille-Yosida Theorem in Hilbert spaces. Application to evolution partial differential equations: heat and wave equation, reaction-diffusion system.
Reference books	H. BREZIS, Analyse fonctionnelle, Theorie et applications, 2e tirage, Masson 1987. H. BREZIS, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer, 2011.
Additional course materials	Lecture notes.

Work schedule				
	Total	Lectures	Hands-on learning	Self-study
Hours	175	56		119
ECTS credits	7			

Teaching methods	
	Lectures.

Expected learning outcomes	
Knowledge and understanding	Acquiring fundamental concepts and results in the setting of functional spaces and operator theory. Acquiring main tools and proof techniques.
Applying knowledge and understanding	The acquired theoretical knowledge finds many applications in several aspects of mathematics, including partial differential equations and related models.

Making judgements	Ability to analyze the consistency of the logical arguments used in a proof, the problem-solving skills and the ability to choose suitable mathematical tools consistent with the theoretical knowledge.
Communication skills	Acquiring the mathematical language and the formalism necessary to read and understand textbooks, to explain the acquired knowledge, and to describe, analyze and solve problems.
Learning skills	Acquiring suitable learning methods, supported by consultation of texts and by solving exercises and problems related to the contents of the course.

Assessment and feedback	
Assessment methods	Oral test
Evaluation criteria	• *Knowledge and understanding*: it will be evaluated the acquisition of the fundamental concepts and results in the setting of functional spaces and operator theory and the acquisition of the main tools and of the proof techniques • *Applying knowledge and understanding*: it will be evaluated the acquired the oretical knowledge in several applications • *Making judgements*: it will be evaluated the ability to analyze the consistency of the logical arguments used in a proof, the problem solving skills and the ability to choose the suitable mathematical tools consistent with the theoretical knowledge • *Communication skills*: it will be evaluated the acquisition of the mathematical language together with the formalism and the preciseness necessary to explain the acquired knowledge, and to describe, analyze and solve problems. • *Learning skills*: it will be evaluated the acquisition of suitable learning methods, supported by consultation of texts and by solving exercises and problems related to the contents of the course.
Grading policy	The exam is sufficient with 18/30 mark.

Additional information	