UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

CONSIGLIO INTERCLASSE IN MATEMATICA

General information	
Academic subject	***Fourier Analysis and Potential theory***
Degree course	*Mathematics*
Academic Year	*2021-22*
European Credit Transfer and Accumulation System (ECTS)	7
Language	*Italian*
Academic calendar (starting and ending date)	*I Semester*
Attendance	*Strongly recommended*

Professor/ Lecturer	
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Tutoring (time and day)	On appointment by e-mail

Syllabus	
Learning Objectives	*Acquiring language and techniques of modern analysis, especially interpolation theorems, maximal functions, Lp-bounded operators, Riesz potential theory, singular integral operators, multipliers theorems, application to linear and semilinear evolution equations; homogeneous Lie groups and sublaplacians, Heisenberg group and the Kohn Laplacian, Haar measures.*
Course prerequisites	*Mathematical knowledge which usually is acquired during a degree of L–35 class. Especially: classical analysis of one and several variables, general topology, linear algebra, Lebesgue measure and integration theory.*
Contents	*1. Fourier Analysis* *Basics on Lp spaces, convolution product, approximation of identity, Fourier transform, Schwartz space and tempered distribution space. Weak (p,q) convergence. Marcinkiewicz interpolation theorem. Hardy-Littlewood maximal function. Diadic maximal function. Calderón-Zygmund decomposition. Poisson kernels, P.V. 1/x, Hilbert transform. Riesz-Kolmogorov theorem. Multipliers. Singular integral operators. Fourier transform of P.V. Ω(x)/|x|n. Method of rotations. Riesz transforms. Riesz and Bessel potentials, fractional Soboleve spaces. Hardy-Littlewood-Sobolev theorem and Sobolev embedding theorems. Calderón-Zygmund theorem. Pseudo-differential operators. Real Hardy spaces Hp, atomic decomposition, BMO, operators in Hardy spaces and in BMO. Weighted inequalities with A1 and Ap weights. Paley-Littlewood decomposition. Mikhlin-Hörmander multiplier theorems. Results for parameter-dependent multiplier theorems and application to evolution equations.* *2. Analysis on homogeneous Lie groups* *Homogeneous Lie groups and sublaplacians. Heisenberg group and the Kohn Laplacian. Homogeneous norms. Haar measures. Convolution on groups. Lp-weak spaces and functional inequalities. Fundamental solution for sublaplacians. Representation formulas. Maximum principle. Hardy-Littlewood-Sobolev theorem for sublaplacians.*
Books and bibliography	*J. Duoandikoetxea, Fourier Analysis, Graduate Studies in Mathematics, Vol 29, AMS, 2000.* *M.R. Ebert, M. Reissig, Methods for Partial Differential Equations, Birkhäuser*

	Basel, 2018. *L. Grafakos, Classical Fourier analysis. Third edition. Graduate Texts in Mathematics, 249. Springer, New York, 2014*
Additional materials	*Lecture notes*

Work schedule			
Total	Lectures	Hands on (Laboratory, working groups, seminars, field trips)	Out-of-class study hours/ Self-study hours
Hours			
175	*60*	*0*	*115*
ECTS			
7	*7*	*0*	

Teaching strategy	
	Lectures, supported by slides and lecture notes.
Expected learning outcomes	
Knowledge and understanding on:	○ Acquiring fundamental concepts in advanced modern real analysis and Fourier Analysis ○ Acquiring mathematical proof techniques and learn the applications to linear and semilinear partial differential equations.
Applying knowledge and understanding on:	○ The acquired theoretical knowledge is useful in large part of mathematics and its applications.
Soft skills	• *Making informed judgments and choices* ○ Ability to analyze the consistency of the logical arguments used in a proof ○ Problem solving skills should be supported by the capacity in evaluating the consistency of the found solution with the theoretical knowledge. ○ Find the most elegant, short, formally correct and complete strategies to solve exercises • *Communicating knowledge and understanding* ○ Students should acquire the mathematical language and formalism that are necessary to read and comprehend textbooks, to expound the acquired knowledge, and to describe, analyse and solve problems. • *Capacities to continue learning* ○ Acquiring suitable learning methods, supported by text consultation and by solving the exercises and questions periodically suggested during the course.

Assessment and feedback	
Methods of assessment	*Oral test.*
Evaluation criteria	• *Knowledge and understanding* ○ Correct and complete discussion ○ Appropriate use of the mathematical tools in the proofs ○ Correct answer to the questions during the oral test • *Applying knowledge and understanding* ○ Correct tools used to completely solve the problems • *Autonomy of judgment* ○ Best tools to solve the question or problem proposed • *Communication skills* ○ Formally correct mathematical language ○ Clarity of the exposition and of the answers

	• *Capacities to continue learning* ○ Correct solutions to the proposed exercises
Criteria for assessment and attribution of the final mark	*The exam is sufficient with 18/30 mark.*
Additional information	