UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

CONSIGLIO INTERCLASSE IN MATEMATICA

COURSE OF STUDY	**THREE-YEAR BACHELOR PROGRAMME IN MATHEMATICS**
ACADEMIC YEAR	**2023-2024**
ACADEMIC SUBJECT	**NONLINEAR ANALYSIS**

General information	
Programme year	Third
Term	Second semester (February 26, 2024 – May 31, 2024)
European Credit Transfer and Accumulation System credits (ECTS)	7
SSD	MAT/05 – Mathematical Analysis
Language	Italian
Mode of attendance	Not mandatory

Lecturer	
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Office hours	Wednesday 15:30-17:30 and by appointment via email

Work schedule				
	Total	Lectures	Hands-on learning	Self-study
Hours	175	56		119
ECTS credits	7	7		

Learning objectives	
	Acquiring instruments of Functional Analysis and an introduction to variational methods and to advanced techniques of Nonlinear Analysis. Furnishing applications to the study of linear and nonlinear variational problems coming from Physics, Geometry and Applied Sciences. In particular proving some abstract existence theorems in Critical Point Theory and studying some nonlinear partial differential equations, whose solutions correspond to critical points of a suitable functional defined on a Banach space.

Course prerequisites	
	In addition to the mathematical knowledge which usually is acquired during the first two years of a degree of L–35 class, they are required language and techniques of modern analysis such as basic theory of Hilbert spaces and Lp spaces.

Syllabus	
Course contents	Variational problems in finite dimension and in infinite dimension. Historical references to classical variational problems. Dido problem. Brachistochrone problem. Fermat's problem. Dirichlet principle (Riemann). Weierstrass counterexample. Function spaces. Background knowledge on spaces of C^k

	functions and on Lebesgue spaces. Elements of distribution theory. The space L^1_loc. Distributions associated with a function of L^1_loc. Dirac delta. Derivative of a distribution. Interpretation of the Dirac Delta as a limit of distributions. Mollifiers. Higher order derivative of a distribution associated with a function L^1_loc. Weak convergence. Sobolev spaces and their main properties. Sobolev Embeddings. Critical exponent. Poincarè Inequality. Linear problems. Elements of spectral theory. Symmetric and self-adjoint operators. Friedrichs extension. Self-adjoint extension and its spectral properties for the Laplace operator with homogeneous Dirichlet boundary conditions. Weak solutions of elliptic boundary value problems. Regularity theorems. Differential Calculus in Banach spaces. Fréchet and Gâteaux derivatives. Theorem of Total Differential. Properties and examples of differentiable functionals. Higher order derivatives. Critical points and local extrema. Caratheodory functions. Nemytskii operators on Sobolev spaces. Theorems on smoothness of Nemytskii operators on L^p spaces. Theorems on smoothness of Nemytskii operators on Sobolev type space. Weak solutions and variational principles for some nonlinear differential problems. Hamilton's principle of least action. Fermat Theorem in Banach spaces. Weierstrass Theorem in Banach spaces. Convexity. Ekeland's variational principle. The Palais-Smale condition and its variants. Existence Theorem of a minimum point for functionals in Banach space. Applications to the search of solutions to nonlinear partial differential equations. Critical points of functionals on manifolds and some problems with constraints: nonlinear eigenvalue problems. Variational problems associated to unbounded functionals. Deformation Theorems. Mountain Pass Theorem. Three Solutions Theorem. Applications to nonlinear elliptic equations on bounded domains with Dirichlet boundary conditions. Pohozaev Identity. Critical elliptic problems on star-shaped domains. Nonlinear Schrodinger Equations in Quantum Mechanics. Nonlinear elliptic problem in the entire space. The Sobolev space H^1(\R^N). Lack of compactness. Subcritical Sobolev inequalities. Brezis-Lieb Lemma. Group Action. Principle of Symmetric Criticality of Palais.
Reference books	• A. Ambrosetti & G. Prodi, "A Primer of Nonlinear Analysis", Cambridge University Press, Cambridge, 1993 • M. Badiale, E. Serra, "Semilinear Elliptic Equations for Beginners", Springer-Verlag 2010 • H. Brezis, "Functional Analysis, Sobolev Spaces and Partial Differential Equations", Springer, New York, 2011 • D. Costa, "An Invitation to Variational Methods in Differential Equations", Birkhäuser, Basel, 2007 • J. Mawhin, M. Willem, "Critical Point Theory and Hamiltonian Systems", Springer-Verlag, Berlin, 1989 • M. Struwe, "Variational Methods. Applications to Nonlinear Partial Differential Equations and Hamiltonian Systems" (4th Ed.), Ergeb. Math. Grenzgeb. (4) 34, Springer–Verlag, Berlin, 2008
Additional course materials	
Repository	

Expected learning outcomes	
Knowledge and understanding	Acquiring fundamental concepts of variational methods, their related proof

	techniques and how to apply them for studying some differential equations.
Applying knowledge and understanding	Applying the acquired variational methods in the study of nonlinear differential equations which describe some classical problems in Geometry and in Mathematical Physics.
Soft skills	*Making judgements*: Ability to analyze the consistency of the logical arguments used in a proof. Problem solving skills should be supported by the capacity in evaluating the correct methods required for studying nonlinear differential equations with variational structure.
	Communication skills: Students should acquire the mathematical language and formalism necessary to read and comprehend textbooks, to explain the acquired knowledge, and to describe, analyze and solve variational problems.
	Learning skills: Acquiring suitable learning methods, supported by text consultation and by solving some nonlinear model differential equations.

Teaching methods	
	The mode of delivery of teaching is frontal. Lessons will be held in presence.

Assessment	
Assessment methods	The exam consists of an oral test on a main theoretical topic of the course contents, chosen independently by the student. An application to the study of nonlinear variational problems is required.
Evaluation criteria	• *Knowledge and understanding:* mastering and deep understanding of the main theoretical course contents. • *Applying knowledge and understanding:* solving nonlinear differential problems by means of variational techniques- • *Making judgement:* approaching notions in a critical way. • *Communication skills*: mastering the language of Nonlinear Analysis and of Critical Point Theory. • *Learning skills:* organizing knowledge and autonomous learning.
Grading policy	The final mark is given out of thirty. The exam is considered passed if the final grade is greater than or equal to 18/30. The evaluation of the oral exam is based on the achievement of the learning objectives. To achieve a high evaluation, the student must have developed independent judgement, adequate understanding of the main topics of the course and presentation skills. The Praise is given in case of further study of some program topic.

Further information	
	Attendance is strongly recommended.