UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

CONSIGLIO INTERCLASSE IN MATEMATICA

General information	Academic year 2022-2023
Academic subject	**Nonlinear Analysis**
Degree programme	Mathematics
Programme year	Third
Term	Second semester (February 27, 2023 – May 26, 2023)
European Credit Transfer and Accumulation System credits (ECTS)	7
Language	Italian
Attendance	Not compulsory

Lecturer	
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Syllabus	
Learning objectives	Acquiring instruments of Functional Analysis and an introduction to variational methods and to advanced techniques of Nonlinear Analysis. Furnishing applications to the study of linear and nonlinear variational problems coming from Physics, Geometry and Applied Sciences. In particular proving some abstract existence theorems in Critical Point Theory and studying some nonlinear partial differential equations, whose solutions correspond to critical points of a suitable functional defined on a Banach space.
Course prerequisites	In addition to the mathematical knowledge which usually is acquired during the first two years of a degree of L–35 class, they are required language and techniques of modern analysis such as basic theory of Hilbert spaces and Lp spaces.
Course contents	Function spaces. Background knowledge on spaces of C^k functions and on Lebesgue spaces. Elements of distribution theory. Weak convergence. Sobolev spaces and their main properties. Sobolev Embeddings. Critical exponent. Poincarè Inequality. Linear problems. Elements of spectral theory. Symmetric and self-adjoint operators. Friedrichs extension. Self-adjoint extension and its spectral properties for the Laplace operator with homogeneous Dirichlet boundary conditions. Weak solutions of elliptic boundary value problems. Regularity theorems. Differential Calculus in Banach spaces. Fréchet and Gâteaux derivatives. Theorem of Total Differential. Properties and examples of differentiable functionals. Higher order derivatives. Critical points and local extrema. Fermat Theorem in Banach spaces. Weierstrass Theorem in Banach spaces. Nonlinear differential problems. Nemytskii operators on Sobolev spaces. Weak solutions and variational principles for some nonlinear differential problems. Hamilton's principle of least action. Weierstrass Theorem and existence of weak solutions. Applications to semilinear elliptic equations. Ekeland's variational principle. Critical points of functionals on manifolds and some problems with constraints: non-homogeneous elliptic problems, dynamical systems on manifolds and their trajectories joining two fixed

	points, nonlinear eigenvalue problems. Variational problems with unbounded functionals. The Palais-Smale condition and its variants. Deformation Theorems. Mountain Pass Theorem. Three Solutions Theorem. Applications to some nonlinear differential equations on bounded and unbounded domains. Pohozaev Identity. Lack of compactness. Group Action. Principle of symmetric criticality. Subcritical Sobolev inequalities. Brezis-Lieb Lemma. Nonlinear Schrodinger Equations in Quantum Mechanics.
Reference books	A. Ambrosetti & G. Prodi, "A Primer of Nonlinear Analysis", Cambridge University Press, Cambridge, 1993 M. Badiale, E. Serra, "Semilinear Elliptic Equations for Beginners", Springer-Verlag 2010 H. Brezis, "Functional Analysis, Sobolev Spaces and Partial Differential Equations", Springer, New York, 2011 D. Costa, "An Invitation to Variational Methods in Differential Equations", Birkhäuser, Basel, 2007 J. Mawhin, M. Willem, "Critical Point Theory and Hamiltonian Systems", Springer-Verlag, Berlin, 1989 M. Struwe, "Variational Methods. Applications to Nonlinear Partial Differential Equations and Hamiltonian Systems" (4th Ed.), Ergeb. Math. Grenzgeb. (4) 34, Springer–Verlag, Berlin, 2008
Additional course materials	

Work schedule				
	Total	Lectures	Hands-on learning (recitations/laboratories /seminars/other)	Self-study
Hours	175	56		119
ECTS credits	7	7		

Teaching methods	

Expected learning outcomes	
Knowledge and understanding	Acquiring fundamental concepts of variational methods, their related proof techniques and how to apply them for studying some differential equations.
Applying knowledge and understanding	Applying the acquired variational methods in the study of nonlinear differential equations which describe some classical problems in Geometry and in Mathematical Physics.
Making judgements	Ability to analyze the consistency of the logical arguments used in a proof. Problem solving skills should be supported by the capacity in evaluating the correct methods required for studying nonlinear differential equations with variational structure
Communication skills	Students should acquire the mathematical language and formalism necessary to read and comprehend textbooks, to explain the acquired knowledge, and to describe, analyze and solve variational problems.
Learning skills	Acquiring suitable learning methods, supported by text consultation and by solving some nonlinear model differential equations.

Assessment and feedback	
Assessment methods	Oral exam starting from the presentation of a chosen topic
Evaluation criteria	• *Knowledge and understanding*: mastering and deep understanding of the main theoretical course contents;

	• *Applying knowledge and understanding*: solving nonlinear differential problems by means of variational techniques; • *Making judgements*: approaching notions in a critical way; • *Communication skills*: mastering the language of Nonlinear Analysis and of Critical Point Theory: • *Learning skills*: organizing knowledge and autonomous learning.
Grading policy	The final grade, out of thirty, is assigned on the basis of an oral exam. The exam is passed if the final grade is greater than or equal to 18/30.

Additional information	
	Attendance is strongly recommended.