General information		Academic year 2022-2023
Academic subject	**Mathematical Analysis 4**	
Degree programme	L 35 - Mathematics	
Programme year	Second	
Term	Second semester (February 27, 2023 – May 26, 2023)	
European Credit Transfer and Accumulation System credits (ECTS)	8	
Language	Italian	
Attendance	Discretionary, but strongly recommended	

Lecturers		
Name and surname	Anna Maria Candela (instructor of record)	Elvira Mirenghi
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Department and office	Department of Mathematics, room 20 second floor	Department of Mathematics, room 29 second floor
Virtual meeting room	Microsoft Teams, code tpkfion	
Web page	https://www.dm.uniba.it/members/candela	https://www.dm.uniba.it/members/mirenghi
Office hours	In-person or online. Days and times have to be arranged by e-mail.	In-person or online. Days and times have to be arranged by e-mail.

Syllabus	
Learning objectives	This course is aimed at providing further language and techniques of classical Mathematical Analysis, in particular concerning ordinary differential equations and their qualitative behavior, basics on Peano-Jordan measure and Riemannian integration, basics on Lebesgue measure and Lebesgue integration for functions of several variables, basics on differential forms and their applications, basics on both line and surface integrals.
Course prerequisites	In order to understand and be able to apply most of the techniques taught in this course, students have to master the basic knowledge given at the first year and in the first semester of the second year of a degree course in Mathematics. More precisely: classical Mathematical Analysis for real single variable functions (differential and integral calculus), curves and functions in several variables, basics on metric and normed spaces, basics on Linear and Matrix Algebra, basics on Analytic Geometry in the plane and in the space.
Course contents	**Line integrals** Line integral of a function, its properties and its applications. **Basics on Peano-Jordan measure theory and Riemannian integration** Outline on plurintervals in IR^N and Peano-Jordan measurable sets (bounded and unbounded). Properties of Peano-Jordan measure. Measure of product spaces. Outline on Riemann integral in bounded or in unbounded measurable sets. Cylindroid. Integration in product spaces. Barycenter. Normal sets and reduction formulas for multiple integrals. Change of variables for multiple integrals. **Lebesgue measure and integral theory** Lebesgue measurable sets. Properties of the Lebesgue measure. Lebesgue measurable functions and related properties. The Lebesgue integral for simple functions, non-negative measurable functions and absolutely integrable functions. Properties of the Lebesgue integral. Vitali-Lebesgue

	Theorem. Beppo-Levi monotone convergence Theorem. Fatou's Lemma. Lebesgue Dominated Convergence Theorem. **Differential 1-forms** Vector field and differential 1-forms. Exact differential 1-forms, their primitives and related properties. Linear integral of a differential 1-form. Physical meaning: conservative fields, potential, work. Integration criteria for differential 1-forms. Closed differential 1-forms and irrotational fields. Relation between closed and exact differential 1-forms. Closed differential forms in starshaped open domains. Positive homogeneous closed differential 1-forms. Gauss-Green formulas in the plane. Closed differential 1-forms in simply connected open domains. **Ordinary differential equations** Introduction to ordinary differential equations. Ordinary differential equations in normal form and Cauchy problems. Theorem of Peano. Local and global existence and uniqueness theorem for ordinary differential equations in normal form. Prolongability of solutions and maximum range of solutions. The sublinear case. General, particular and singular integrals. Equivalence between differential equations of higher order and suitable systems of first order differential equations. Systems of linear first order differential equations: existence and uniqueness of the solution in large. Wronskian of *n* solutions and its properties. Dimension of the solutions set of a homogeneous system of linear first order differential equations. General integral of a complete linear system of first order differential equations. Lagrange method for the computation of a particular integral. Linear differential *n*-order equations with constant coefficients: characteristic equation. Linearly independent solutions for differential equations with constant coefficients, once known the roots of the characteristic equation. Linear differential *n*-order equations with variable coefficients. Solution methods for the following first order differential equations: separable variables, Manfredi's (or homogeneous), of type y'=f[(ax+by+c)/(a'x+b'y+c')], linear, Bernoulli's, Clairaut's, Lagrange's, with an integrating factor. Second order differential equations which may be reduced to first order differential equations. Lowering the order of a differential equations of order *n* with constant coefficients. Euler's equations. Simple systems of linear differential equations. **Surfaces and surface integrals** Parametric surfaces in IR^3: properties and examples. Tangent plane and normal versor. Regular surfaces. Area of a regular surface. Area of a rotational surface. Surface integral of a function. Oriented surfaces. Flow of a vector field through a surface. Divergence (or Gauss) Theorem in IR^3. Stokes (or rotor) Theorem. **Hypersurfaces in IR^N** k-dimensional manifolds and hypersurfaces in IR^N. Tangent space and normal space to a manifold. Measurement and integration on the k-dimensional submanifolds of IR^N. Divergence Theorem in IR^N.
Reference books	• N. Fusco - P. Marcellini – C. Sbordone, Lezioni di Analisi Matematica Due, Zanichelli Ed., Bologna, 2020. (Chapters 4, 5, 6, 7, 8, 9, 10, 12) • F. Gazzola, Analisi Matematica 2, Società Ed. Esculapio, Bologna 2022 • P. Marcellini – C. Sbordone, Esercitazioni di Analisi Matematica Due, Prima parte, Zanichelli Ed., Bologna, 2018 • P. Marcellini – C. Sbordone, Esercitazioni di Analisi Matematica Due, Seconda parte, Zanichelli Ed., Bologna, 2018

Additional course materials	It is recommended to complete textbooks with notes taken at lesson. The recommended textbook can be replaced by any other books of Mathematical Analysis which cover the topics of the program. If using notes found on internet, a careful check about their author is strongly recommended.

Work schedule				
	Total	Lectures	Hands-on learning (recitations)	Self-study
Hours	200	40	30	130
ECTS credits	8	5	3	

Teaching methods	
	Classroom lectures which include exercises whose purpose is to make the student acquire the ability to apply theoretical concepts. Teaching will take place according to the Academic Senate's resolutions.

Expected learning outcomes	
Knowledge and understanding	Fundamental concepts in the classical Mathematical Analysis and related mathematical proof techniques.
Applying knowledge and understanding	The acquired mathematical knowledge is useful in large part of Mathematics and its applications.
Making judgements	Students have to develop critical thinking so to distinguish between essential and nonessential assumptions, moreover they have to identify the most appropriate mathematical tools for solving a given problem and have to realize the limitations of techniques and methods.
Communication skills	Students have to acquire both language and advanced mathematical formalism so to be able to retrieve useful information from Maths textbooks, to discuss mathematical results and to describe, to analyze and to solve given problems.
Learning skills	Students have to acquire the ability to study and understand mathematical topics, and also to retrieve useful information from Maths textbooks so to apply them for solving problems.

Assessment and feedback	
Assessment methods	The final grade comes from: • a written exam (about two hours long), • an oral exam. The written test is preparatory to the oral exam. Its result will be communicated either by e-mail or through platform ESSE3.
Evaluation criteria	The student must: • solve exercises on line integrals and multiple integrals, • solve exercises on differential 1-forms, • solve exercises on ordinary differential equations/systems and related Cauchy problems, • solve exercises on surface integrals of a function. The oral exam is designed to assess the level of knowledge attained by the student on the theoretical contents of the course. The student has to distinguish between essential and nonessential assumptions, to discuss mathematical notions in a rigorous way, to contextualize mathematical topics.

Grading policy	The final grade, out of thirty, takes into account both the written and the oral part. The exam is passed if the final grade is greater than or equal to 18/30. For taking the oral exam, a student must pass the written test with a grade which is greater than or equal to 15/30.

Additional information	