UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

CONSIGLIO INTERCLASSE IN MATEMATICA

COURSE OF STUDY	**THREE-YEAR BACHELOR PROGRAMME IN MATHEMATICS**
ACADEMIC YEAR	**2023-2024**
ACADEMIC SUBJECT	**MATHEMATICAL ANALYSIS 3**

General information	
Programme year	Second
Term	First semester (September 25, 2023 – December 22, 2023)
European Credit Transfer and Accumulation System credits (ECTS)	8
SSD	MAT/05 – Mathematical Analysis
Language	Italian
Mode of attendance	Not mandatory

Lecturers		
Name and surname	Anna Maria Candela (instructor of record)	Elvira Mirenghi
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Department and office	Department of Mathematics room 20 second floor	Department of Mathematics room 29 second floor
Virtual meeting room	Microsoft Teams, code tpkfion	
Web page	https://www.dm.uniba.it/en/members/candela	https://www.dm.uniba.it/en/members/mirenghi
Office hours	In-person or online. Days and times have to be arranged by e-mail.	In-person or online. Days and times have to be arranged by e-mail.

Work schedule				
	Total	Lectures	Hands-on learning (recitations)	Self-study
Hours	200	48	30	122
ECTS credits	8	6	2	

Learning objectives	
	This course is aimed at providing further language and techniques of classical Mathematical Analysis, in particular about sequences and series of functions, basics of metric and normed spaces, basics on curves, basics on functions in several variables.

Course prerequisites	
	In order to understand and be able to apply most of the techniques taught in this course, students have to master the basic knowledge given at the first year of a degree course in Mathematics. In particular, classical Mathematical Analysis for real single variable functions (differential and integral calculus), basics of Linear and Matrix Algebra, Analytic Geometry.

Syllabus	
Course contents	***Sequences and series of functions*** Sequences of functions: pointwise and uniform convergence. Theorem on

	the interchange of limits. Theorem on the continuity of the limit function. Cauchy criterion for uniform convergence. Theorem on the interchange of limit and integral. Theorem on the interchange of limit and derivative. Series of functions: pointwise, uniform and absolute convergence. Cauchy criterion for uniform convergence of series. Theorem on the term-by-term integration. Theorem on term-by-term differentiation. Properties of the sum function. Power series and set of convergence. Radius of convergence and its characterization. Cauchy-Hadamard Theorem. D'Alembert Theorem. Abel Theorem. Power series obtained by integration or derivation and their convergence radius. Taylor series and analytic functions. Sufficient condition for the expansion of a function in Taylor series. The binomial series. Noteworthy Taylor series. Fourier series. Dirichlet's criterion on the pointwise convergence of a Fourier series. Theorem on the uniform convergence of a Fourier series. ***Metric spaces, normed spaces, the Euclidean space IR^N*** Metric spaces and distances. Equivalent metrics. Topology of a metric space: open balls, neighborhoods, open and closed subsets. Interior, limit, boundary and isolated points of a set. Connected sets. Bounded sets and diameter. Convergent sequences and Cauchy sequences. Complete metric spaces. Compact sets. Limits and continuity in metric spaces. Lipschitz-continuous mappings and Banach fixed-point theorem in metric spaces. Vector spaces. Linearly independent vectors and bases. Dual space. The linear space IR^N and its standard basis. Linear functionals on IR^N. Linear maps from IR^N to IR^m and matrices. Normed spaces and Banach spaces. Equivalent norms. Noteworthy normed spaces. Norm induced metric and norm associated to an inner product. Bounded functions. The linear space IR^N: noteworthy norms and inner product. Young, Hölder and Minkowski inequalities. Compact subsets of IR^N and Heine-Borel Theorem. The (generalized) Weierstrass theorem. Uniformly continuous mappings. The (generalized) Cantor theorem. Line segments, polygons, convex sets, star-shaped sets, connected sets in IR^N and their characterization. Limits of functions in IR^N. Continuous functions: properties and main theorems. ***Curves*** Curves and their parametric representations. Simple, open and closed curves. Equivalent curves. Regular and piecewise regular curves. Tangent to a curve in a regular point. Rectifiable curves and their length. Rectifiability of a piecewise regular curve. Invariance with respect to change of variables. Arc-length parametrization. Curvature of a plane curve. ***Differential calculus for functions in several variables*** Real functions in several variables: partial derivatives and their properties. Gradient vector. Higher order partial derivatives. Schwarz theorem and Hessian matrix. Directional derivative or Gateaux derivative. Differentiability and the Fréchet derivative. Differentiable functions: properties. Sufficient condition for the differentiability of a function. Chain rule for differentiable functions. Lagrange Mean Value Theorem. Functions with zero gradient. Taylor formulas. Homogeneous functions and Euler's homogeneous functions theorem. Functions defined by integrals: continuity and differentiability. Vector-valued functions from IR^N to IR^m and the Jacobian matrix. Differentiability and related theorems. The higher-dimensional chain rule.

	Square matrices and their eigenvalues. Quadratic forms and symmetric matrices. Definite, semidefinite matrices and their characterizations. Local extrema for real functions in several variables. Fermat Theorem. Necessary condition of the second order. Second partial derivative test for local extrema. Constrained and absolute extrema. ***The implicit function theorem*** Implicit function problem. Implicit function theorem and regularity results in two or more variables. System of equations, choosing dependent and independent variables and generalized Implicit Function Theorem. Constrained extrema and the Method of Lagrange Multipliers.
Reference books	• N. Fusco - P. Marcellini – C. Sbordone, Lezioni di Analisi Matematica Due, Zanichelli Ed., Bologna, 2020. (Capitoli 1, 2, 3, 6, 11) • F. Gazzola, Analisi Matematica 2, Società Ed. Esculapio, Bologna 2022 • P. Marcellini – C. Sbordone, Esercitazioni di Analisi Matematica Due, Prima parte, Zanichelli Ed., Bologna, 2018 • P. Marcellini – C. Sbordone, Esercitazioni di Analisi Matematica Due, Seconda parte, Zanichelli Ed., Bologna, 2018
Additional course materials	It is recommended to complete textbooks with notes taken at lesson. The recommended textbook can be replaced by any other books of Mathematical Analysis which cover the topics of the program. If using notes found on internet, a careful check about their author is strongly recommended.
Repository	Notes of both professors are available upon request.

Expected learning outcomes	
Knowledge and understanding	Fundamental concepts in the classical Mathematical Analysis and related mathematical proof techniques.
Applying knowledge and understanding	The acquired mathematical knowledge is useful in large part of Mathematics and its applications.
Soft skills	*Making judgements*: Students have to develop critical thinking so to distinguish between essential and nonessential assumptions, moreover they have to identify the most appropriate mathematical tools for solving a given problem and have to realize the limitations of techniques and methods.
	Communication skills: Students have to acquire both language and advanced mathematical formalism so to be able to retrieve useful information from Maths textbooks, to discuss mathematical results and to describe, to analyze and to solve given problems.
	Learning skills: Students have to acquire the ability to study and understand mathematical topics, and also to retrieve useful information from Maths textbooks so to apply them for solving problems.

Teaching methods	
	Classroom lectures which include exercises whose purpose is to make students acquire the ability to apply theoretical concepts. Teaching will take place in-person, anyway according to the Academic Senate's resolutions.

Assessment	
Assessment methods	The final grade comes from: • a written exam (about two hours long), • an oral exam. The written test is preparatory to the oral exam. Its result will be

	communicated either by e-mail or through platform ESSE3.
Evaluation criteria	• *Knowledge and understanding*: The oral exam is designed to assess the level of knowledge attained for mastering definitions and theoretical results in the program. • *Applying knowledge and understanding*: students have to be able to apply the acquired theoretical knowledge for: solving exercises on sequences and series of functions, solving exercises on curves, studying real functions in several variables (limits and local, constrained, global minimum and maximum points), studying implicit fuctions. • *Making judgement*: students have to be able to distinguish between essential and nonessential assumptions, with a critical approach and following a logical thread in proofs, and also to choose the right mathematical tools and techniques for dealing with complex mathematical problems. • *Communication skills*: students have to be able to discuss mathematical notions in a rigorous way. • *Learning skills*: students have to be able to contextualize mathematical topics and eventually extend their knowledge independently.
Grading policy	The final grade, out of thirty, takes into account both the written and the oral part. The exam is passed if the final grade is greater than or equal to 18/30. For taking the oral exam, a student must pass the written test with a grade which is greater than or equal to 15/30.

Further information	
	Attendance is strongly recommended.