

General information		Academic year 2022-2023
Academic subject	<b>Mathematical Analysis 3</b>	
Degree programme	L 35 - Mathematics	
Programme year	Second	
Term	First semester (September 26, 2022 – December 22, 2022)	
European Credit Transfer and Accumulation System credits (ECTS)	8	
Language	Italian	
Attendance	Discretionary, but strongly recommended	

Lecturers		
Name and surname	Anna Maria Candela (instructor of record)	Elvira Mirenghi
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Department and office	Department of Mathematics, room 20 second floor	Department of Mathematics, room 29 second floor
Virtual meeting room	Microsoft Teams, code tpkfion	
Web page	<a href="https://www.dm.uniba.it/members/candela">https://www.dm.uniba.it/members/candela</a>	<a href="https://www.dm.uniba.it/members/mirenghi">https://www.dm.uniba.it/members/mirenghi</a>
Office hours	In-person or online. Days and times have to be arranged by e-mail.	In-person or online. Days and times have to be arranged by e-mail.

Syllabus	
<b>Learning objectives</b>	This course is aimed at providing further language and techniques of classical Mathematical Analysis, in particular about sequences and series of functions, basics of metric and normed spaces, basics on curves, basics on functions in several variables.
<b>Course prerequisites</b>	In order to understand and be able to apply most of the techniques taught in this course, students have to master the basic knowledge given at the first year of a degree course in Mathematics. In particular, classical Mathematical Analysis for real single variable functions (differential and integral calculus), basics of Linear and Matrix Algebra, Analytic Geometry.
<b>Course contents</b>	<p><b>Sequences and series of functions</b>            Sequences of functions: pointwise and uniform convergence. Theorem on the interchange of limits. Theorem on the continuity of the limit function. Cauchy criterion for uniform convergence. Theorem on the interchange of limit and integral. Theorem on the interchange of limit and derivative. Series of functions: pointwise, uniform and absolute convergence. Cauchy criterion for uniform convergence of series. Theorem on the term-by-term integration. Theorem on term-by-term differentiation. Properties of the sum function. Power series and set of convergence. Radius of convergence and its characterization. Cauchy-Hadamard Theorem. D'Alembert Theorem. Abel Theorem. Power series obtained by integration or derivation and their convergence radius. Taylor series and analytic functions. Sufficient condition for the expansion of a function in Taylor series. The binomial series. Noteworthy Taylor series. Fourier series. Dirichlet's criterion on the pointwise convergence of a Fourier series. Theorem on the uniform convergence of a Fourier series.</p> <p><b>Metric spaces, normed spaces, the Euclidean space <math>\mathbb{R}^N</math></b>            Metric spaces and distances. Equivalent metrics. Topology of a metric space: open balls, neighborhoods, open and closed subsets. Interior, limit,</p>



	<p>boundary and isolated points of a set. Connected sets. Bounded sets and diameter. Convergent sequences and Cauchy sequences. Complete metric spaces. Compact sets. Limits and continuity in metric spaces. Lipschitz-continuous mappings and Banach fixedpoint theorem in metric spaces.</p> <p>Vector spaces. Linearly independent vectors and bases. Dual space. The linear space <math>\mathbb{R}^N</math> and its standard basis. Linear functionals on <math>\mathbb{R}^N</math>. Linear maps from <math>\mathbb{R}^N</math> to <math>\mathbb{R}^m</math> and matrices. Normed spaces and Banach spaces. Equivalent norms. Noteworthy normed spaces. Norm induced metric and norm associated to an inner product. Bounded functions. The linear space <math>\mathbb{R}^N</math>: noteworthy norms and inner product. Young, Hölder and Minkowski inequalities. Compact subsets of <math>\mathbb{R}^N</math> and Heine-Borel Theorem. The (generalized) Weierstrass theorem. Uniformly continuous mappings. The (generalized) Cantor theorem. Line segments, polygons, convex sets, star-shaped sets, connected sets in <math>\mathbb{R}^N</math> and their characterization. Limits of functions in <math>\mathbb{R}^N</math>. Continuous functions: properties and main theorems.</p> <p><b>Curves</b></p> <p>Curves. Simple, open and closed curves. Parametric representation of a curve and equivalent curves. Regular curves and piecewise regular curves. Tangent to a curve in a regular point. Rectifiable curves and their length. Rectifiability of a piecewise regular curve. Invariance with respect to change of variables. Arc-length parametrization. Curvature of a plane curve.</p> <p><b>Differential calculus for functions in several variables</b></p> <p>Real functions in several variables: partial derivatives and their properties. Gradient vector. Higher order partial derivatives. Schwarz theorem and Hessian matrix. Directional derivative or Gateaux derivative. Differentiability and the Fréchet derivative. Differentiable functions: properties. Sufficient condition for the differentiability of a function. Chain rule for differentiable functions. Lagrange Mean Value Theorem. Functions with zero gradient. Taylor formulas. Homogeneous functions and Euler's homogeneous functions theorem. Functions defined by integrals: continuity and differentiability. Vector-valued functions from <math>\mathbb{R}^N</math> to <math>\mathbb{R}^m</math> and the Jacobian matrix. Differentiability and related theorems. The higher-dimensional chain rule. Square matrices and their eigenvalues. Quadratic forms and symmetric matrices. Definite, semidefinite matrices and their characterizations. Local extrema for real functions in several variables. Fermat Theorem. Necessary condition of the second order. Second partial derivative test for local extrema. Constrained and absolute extrema.</p> <p><b>The implicit function theorem</b></p> <p>Implicit function problem. Implicit function theorem and regularity results in two or more variables. System of equations, choosing dependent and independent variables and generalized Implicit Function Theorem. Constrained extrema and the Method of Lagrange Multipliers.</p>
<b>Reference books</b>	<ul style="list-style-type: none"><li>• N. Fusco - P. Marcellini – C. Sbordone, <i>Lezioni di Analisi Matematica Due</i>, Zanichelli Ed., Bologna, 2020. (Chapters 1, 2, 3, 6, 11)</li><li>• F. Gazzola, <i>Analisi Matematica 2</i>, Società Ed. Esculapio, Bologna 2022</li><li>• P. Marcellini – C. Sbordone, <i>Esercitazioni di Analisi Matematica Due</i>, Prima parte, Zanichelli Ed., Bologna, 2018</li><li>• P. Marcellini – C. Sbordone, <i>Esercitazioni di Analisi Matematica Due</i>, Seconda parte, Zanichelli Ed., Bologna, 2018</li></ul>
<b>Additional course materials</b>	It is recommended to complete textbooks with notes taken at lesson. The recommended textbook can be replaced by any other books of Mathematical Analysis which cover the topics of the program. If using notes

	found on internet, a careful check about their author is strongly recommended.
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Work schedule				
	Total	Lectures	Hands-on learning (recitations)	Self-study
<b>Hours</b>	200	40	30	130
<b>ECTS credits</b>	8	5	3	

Teaching methods	
	Classroom lectures which include exercises whose purpose is to make the student acquire the ability to apply theoretical concepts. Teaching will take place according to the Academic Senate's resolutions.

Expected learning outcomes	
<b>Knowledge and understanding</b>	Fundamental concepts in the classical Mathematical Analysis and related mathematical proof techniques.
<b>Applying knowledge and understanding</b>	The acquired mathematical knowledge is useful in large part of Mathematics and its applications.
<b>Making judgements</b>	Students have to develop critical thinking so to distinguish between essential and nonessential assumptions, moreover they have to identify the most appropriate mathematical tools for solving a given problem and have to realize the limitations of techniques and methods.
<b>Communication skills</b>	Students have to acquire both language and advanced mathematical formalism so to be able to retrieve useful information from Maths textbooks, to discuss mathematical results and to describe, to analyze and to solve given problems.
<b>Learning skills</b>	Students have to acquire the ability to study and understand mathematical topics, and also to retrieve useful information from Maths textbooks so to apply them for solving problems.

Assessment and feedback	
Assessment methods	The final grade comes from: <ul style="list-style-type: none"> <li>• a written exam (about two hours long),</li> <li>• an oral exam.</li> </ul> The written test is preparatory to the oral exam. Its result will be communicated either by e-mail or through platform ESSE3.
Evaluation criteria	The student must: <ul style="list-style-type: none"> <li>• solve exercises on sequences and series of functions,</li> <li>• solve exercises on curves,</li> <li>• study real functions in several variables: limits and local, constrained, global minimum and maximum points,</li> <li>• study implicit functions.</li> </ul> The oral exam is designed to assess the level of knowledge attained by the student on the theoretical contents of the course. The student has to distinguish between essential and nonessential assumptions, to discuss mathematical notions in a rigorous way, to contextualize mathematical topics.
Grading policy	The final grade, out of thirty, takes into account both the written and the oral part. The exam is passed if the final grade is greater than or equal to 18/30.



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	For taking the oral exam, a student must pass the written test with a grade which is greater than or equal to 15/30.
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Additional information	