UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

CONSIGLIO INTERCLASSE IN MATEMATICA

COURSE OF STUDY	**THREE-YEAR BACHELOR PROGRAMME IN MATHEMATICS**
ACADEMIC YEAR	**2023-2024**
ACADEMIC SUBJECT	**GEOMETRY 3**

General information	
Programme year	Second
Term	First semester (September 25, 2023 – December 22, 2023)
European Credit Transfer and Accumulation System credits (ECTS)	8
SSD	MAT/03 – Geometry
Language	Italian
Mode of attendance	Not mandatory

Lecturers		
Name and surname	Francesco Bastianelli (instructor of record)	Donatella Iacono
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Department and office	Department of Mathematics room 18 second floor	Department of Mathematics room 11 third floor
Virtual meeting room		
Web page	https://sites.google.com/site/francescobastianelli/	https://www.donatellaiacono.it
Office hours	in order to schedule an appointment, please contact the lecturer by email	please visit the web page https://www.donatellaiacono.it

Work schedule				
	Total	Lectures	Hands-on learning (recitations)	Self-study
Hours	200	48	30	122
ECTS credits	8	6	2	

Learning objectives	
	Acquiring the basic concepts in Projective Geometry and in the theory of conics and quadrics.

Course prerequisites	
	Mathematical knowledge which is usually acquired during the first year of the degree in Mathematics. Especially: linear algebra, affine and Euclidean spaces.

Syllabus	
Course contents	**Projective spaces.** The projective space P(V) associated to a vector space V. Projective spaces. Coordinate systems, homogeneous coordinates and coordinates changes. Linear subspaces; lines, planes, hyperplanes. Projectively independent points. Points in general position and their characterization. Projective

	Grassmann formula. Intersection of subspaces and projective subspaces joining a finite family of sublinear subspaces in general position. Cartesian and parametric equations of a subspace. Relation between affine and projective geometry. Affine structure on a projective space minus a hyperplane. Projective completion of an affine space. Projective closure of a linear subspace. Relation between the equations of an affine subspace and those of its projective closure. Dual projective space. Duality map. Linear system of hyperplanes containing a given subspace. Duality theorem. Projective transformations. Projective transformation group. Existence and uniqueness theorem for projective transformations. Images of linear subspaces under projective transformations. Projectively equivalent subsets and projective invariants. Relation between affine transformations and projective transformations; canonical extension of an affine transformation to a projective transformation; isomorphism between the group of affine transformation and the group of projective transformation preserving the infinity hyperplane. Projective geometry in one dimension. Cross ratio. Harmonic 4-tuples. Characterization of projective transformations of projective lines as bijections preserving cross ratio. Projective transformations sending an ordered 4-tuple of distinct points in another. Characterization of projectively equivalent 4-tuples of points. Fixed points of a projective transformation. Elliptic, parabolic and hyperbolic projective transformations and their characterization in the complex case and in the real case. Representation of a projective transformation by rational functions. **Projective and affine hypersurfaces.** Ring of polynomials with coefficients belonging to a field. Degree of a polynomial. Irreducible polynomials and factorization. Homogeneous polynomials and their properties. Euler formula. Homogeneization and dehomogeneization of a polynomial. Formal partial derivative of a polynomial. Factorization of a complex polynomial in two variables. Projective algebraic hypersurfaces. Equation, degree and support of a hypersurface. Irreducible components of a hypersurface and their multiplicity. Projectively equivalent hypersurfaces and their properties. Affine algebraic hypersurfaces. Equation, degree and support of a hypersurface. Irreducible components of a hypersurface and their multiplicity. Affinely equivalent hypersurfaces and their properties. Study's Lemma. **Projective quadrics.** Preliminary notions on quadratic forms and symmetric bilinear forms. Notion of projective quadric. Matrix and bilinear form associated to a quadric. Rank of a quadric and index of a real quadric. Projectively equivalent quadrics. Projective classification of real and complex quadrics. Projective classification of quadrics of the real and complex projective line. Intersection between a quadric and a linear subspace. Relative position between a line and a quadric. Singular points of a quadric

	and their characterization. The space of quadrics; linear systems of quadrics. Conjugated points. Polar hyperplane of a non-singular point. Polarity of a non-singular quadric; pole of a hyperplane; conjugated hyperplanes. Tangent hyperplane to a quadric at a non-singular point; characterization in terms of tangent lines; characterization in terms of intersection with the quadric. Tangent lines to a conic; inner and outer points to a conic. Tangent planes to a non-singular quadric surface. Elliptic and hyperbolic points of a non-singular quadric surface; characterization of elliptic and hyperbolic quadrics. Projective classification of real and complex quadric surfaces. **Affine quadrics.** Notion of affine quadric. Matrix associated to an affine quadric. Projective closure of an affine quadric; points at infinity; conic at infinity. Center of a non-singular quadric. Central and non-central quadrics. Coordinates of the center. The center is a center of symmetry. Diametral hyperplanes and their conjugated directions. Affinely equivalent quadrics. Classification of real quadrics. Classification of real non-singular quadrics; ellipsoids, hyperboloids, and paraboloids. Classification of real singular quadrics; cones; elliptic, parabolic, and hyperbolic cylinders. **Euclidean quadrics.** Metric classification of quadrics. Hyperspheres. Principal hyperplanes of a non-singular quadric and their characterization. The set of principal hyperplanes is the disjoint union of linear systems of hyperplanes. Round quadric surfaces. Metric properties of conics. Relative position between a non-singular affine conic and the line at infinity. Asymptotes of a hyperbola and rectangular hyperbolas. Vertices of a conic; axes of a conic and their characterization. Foci of a conic, direttrix, focal axe, eccentricity. **Affine and projective algebraic curves.** Notion of affine algebraic curve. Definition and properties of the intersection multiplicity between a line and an algebraic curve at a point. Multiplicity of a point of a curve; regular and singular points; nodes, cusps, ordinary singularities, m-uple points. Flexes. Tangent lines and tangent cone to a curve at a point. Characterization of singular points. Equation of the tangent line at a regular point. Number of lines in the tangent cone. Characterization of m-uple points. Notion of projective curve. Definition and properties of the intersection multiplicity between a line and an algebraic curve at a point. Multiplicity of a point of a curve; regular and singular points. Tangent lines and tangent cone to a curve at a point. Projective closure of an affine curve, points at infinity, and asymptotes. Characterization of singular points. Equation of the tangent line at a regular point. Characterization of m-uple points.
Reference books	- E. Sernesi: *Geometria 1,* Bollati Boringhieri, 1994. - E. Fortuna, R. Frigerio, R. Pardini: *Geometria proiettiva, problemi risolti e richiami di teoria*, Springer-Verlag, Collana Unitext, 2011. - M. Beltrametti, E. Carletti, D. Gallarati, G. Monti Bragadin: *Lezioni di Geometria analitica e proiettiva*, Bollati Boringhieri, 2003. - M. Berger: *Geometry II*, Universitext, Springer-Verlag, 1987. - E. Casas-Alvero: *Analytic Projective Geometry,* EMS Textbooks in Mathematics, European Mathematical Society (EMS), 2014.

Additional course materials	Further information will be available at https://sites.google.com/site/francescobastianelli/teaching https://www.donatellaiacono.it/teaching.html and further materials will be uploaded on Microsoft Teams (team code: 071qqa5).
Repository	

Expected learning outcomes	
Knowledge and understanding	Acquiring fundamental concepts about classical geometrical topics using a modern language.
Applying knowledge and understanding	The acquired knowledge has a wide spectrum of applications, both in the field of pure mathematics and other scientific disciplines, for example in computer science (3D graphics, design, robotics, computer vision, etc.).
Soft skills	*Making judgements*: ability in developing new methods which are useful in problem solving.
	Communication skills: acquiring the mathematical language and formalism which are necessary to analyze and solve problems.
	Learning skills: acquiring suitable learning methods and ability of relating the main concepts occurring in various mathematical disciplines.

Teaching methods	
	Lectures and exercise classes.

Assessment	
Assessment methods	Written and oral exam about the topic of the course, to evaluate the understanding of the themes investigated.
Evaluation criteria	• *Knowledge and understanding*: quality and accuracy of the techniques, of the proofs, and of the abstract reasoning based on the topic of the course. • *Applying knowledge and understanding*: ability of apply the techniques and the notions presented in the course in order to solve concrete geometric problems. • *Making judgements*: ability of deciding the accuracy of a formal reasoning and ability of choosing suitable techniques for solving a problem. • *Communication skills*: quality and accuracy of the acquired knowledge and of the reasoning skills. • *Learning skills*: organization of knowledge, critical reasoning, possible additional study of the topics related to the course.
Grading policy	The final assessment is given in the range 18/30 – 30/30 e lode. The exam consists of a written and oral test. The written test consists of a series of exercises. Any exercise correspond to a score and the sum of the scores is 30. In order to take the oral test, it is mandatory to pass the written test by obtaining a score greater than or equal to 18/30. The final assessment is communicated after the oral test and it takes in account also the assessment of the written test. The exam is passed if the assessment is greater than or equal to 18/30. It depends on the completeness, the quality, the accuracy and the precision showed during the exam, concerning the acquired knowledge and ability.

Further information

	Attending classes is strongly recommended.