UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

CONSIGLIO INTERCLASSE IN MATEMATICA

COURSE OF STUDY THREE-YEAR BACHELOR PROGRAMME IN MATHEMATICS

ACADEMIC YEAR 2023-2024

ACADEMIC SUBJECT ALGEBRA 1

General information	
Programme year	Second
Term	First semester (September 25, 2023 – December 22, 2024)
European Credit Transfer and Accumulation System credits (ECTS)	8
SSD	MAT/02 – Algebra
Language	Italian
Mode of attendance	Not mandatory

Lecturer	
Name and surname	Margherita Barile
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Department and office	Department of Mathematics, room 23 second floor
Virtual meeting room	Microsoft Teams – code: **nceeadf**
Web page	https://www.dm.uniba.it/it/members/barile
Office hours	By appointment: a virtual meeting on Microsoft Teams can be requested by e-mail.

Work schedule	Total	Lectures	Hands-on learning (recitations)	Self-study
Hours	200	56	15	129
ECTS credits	8	7	1	

Learning objectives	
	Acquiring a solid knowledge of algebraic structures

Course prerequisites	
	Basic concepts of set theory, including maps, relations, number sets

Syllabus	
Course contents	**Number sets:** The divisibility relation in $\mathbb{Z}$, prime numbers. Euclidean division in $\mathbb{Z}$. The GCD and Bézout's Identity, the Euclidean algorithm. The Fundamental Theorem of Arithmetic. Euclid's theorem on the infinitude of primes. The complex numbers as ordered pairs of real numbers, operations in $\mathbb{C}$, algebraic and trigonometric form, the *n*-th roots of a complex number, the Fundamental Theorem of Algebra. **Algebraic structures and homomorphisms:** Elementary properties of groups, Abelian groups, subgroups, cyclic groups and their generators, order of a periodic element, Lagrange's

	Theorem on the order of elements in a finite Abelian group. Elementary properties of rings, commutative rings, unit rings, invertible element, integral domains, division rings, fields, subrings, subfields. Rings of matrices. Direct product of groups and rings. Homomorphisms, monomorphisms, epimorphisms, isomorphisms, kernel of a homomorphism. **Polynomial rings:** Polynomials in one indeterminate, degree of a polynomial. Operations on polynomials. Polynomials with coefficients in an integral domain, the degree formula. Euclidean division in $K[x]$. The GCD and Bézout's Identity, the Euclidean division algorithm. Roots of a polynomial, Rational root theorem. Irreducible polynomials, factorization, associate polynomials. Algebraically closed fields. Gauss' Theorem and factorizations in $\mathbb{Q}[x]$. Reduction modulo *p*, Eisenstein's irreducibility criterion. Irreducible polynomials in $\mathbb{C}[x]$ and in $\mathbb{R}[x]$. **Quotient structures:** The congruence modulo *n* in $\mathbb{Z}$. The residue class ring $\mathbb{Z}_n$. Linear congruences in $\mathbb{Z}$ and linear equations in $\mathbb{Z}_n$. The group of units of $\mathbb{Z}_n$. The fields $\mathbb{Z}_p$. The Euler function. The Chinese Remainder Theorem. Fermat's little Theorem, the Euler Theorem. The congruence modulo $f(x)$ in $K[x]$. The residue class ring $K[x]/f(x)$ and its units. **Symmetric groups:** The natural action of S_n on $X=\{1,\ldots,n\}$. Orbits and cycles of a permutation. Decomposition of a permutation into disjoint cycles. Parity of a permutation, the alternating group A_n.
Reference books	*Appunti di Algebra 1*, Giulio Campanella (Nuova Cultura) *Algebra*, G.M. Piacentini Cattaneo (Decibel- Zanichelli) *Elementi di Algebra*, S. Franciosi, F. de Giovanni (Aracne Editrice) *Algebra*, I.N. Herstein (Editori Riuniti) *Aritmetica e algebra*, D. Dikranjan, M.S. Lucido (Liguori Editore)
Additional course materials	Material available on line: - Complete lecture notes - Exam sheets - Collections of exercises - Additional course material (historical notes, solved exercises, further remarks)
Repository	https://www.dm.uniba.it/members/barile/homepage/algebra-n-1

Expected learning outcomes	
Knowledge and understanding	Recognizing algebraic structures and their properties
Applying knowledge and understanding	Solving algebraic problems by means of a structural approach
Soft skills	*Making judgements*: Assessing the correctness of numerical results by

	reference to a conceptual framework
	Communication skills: Formulating definitions and abstract arguments in a formally rigorous manner
	Learning skills: Establishing logical connections between different topics

Teaching methods	
	Lectures and exercise sessions

Assessment	
Assessment methods	Written exam (3 hours, exercises) and oral exam
Evaluation criteria	• *Knowledge and understanding*: Logical reasoning and abstraction • *Applying knowledge and understanding*: Solving theoretical problems • *Making judgements*: Approaching notions in a critical way • *Communication skills*: Mastering the algebraic language • *Learning skills*: Organizing knowledge
Grading policy	The grade (18-30) is assigned on the basis of an oral exam, after passing the written exam, according to the attained level of knowledge and skills. 18-21: sufficient 22-25: fair 26-29: good 30: very good 30 e lode: outstanding