Storia e Fondamenti della Matematica
a.a. 2020/2021

Traccia d'esame – Novembre 2021 - 1

L’algebra, nel suo carattere generale e innovatore, apre, nel discorso matematico, una finestra sul mondo, di cui interpreta la - in parte sfuggente - complessità. Alla luce del brano iniziale del saggio *A Treatise of Algebra* (1748) del matematico scozzese Colin Maclaurin, esaminare la precedente affermazione, tramite un’analisi testuale a sfondo storico ed incentrata sui seguenti aspetti:

- il rapporto tra aritmetica ed algebra;
- il ruolo linguistico e logico di simboli e segni;
- le categorie di opposti come criteri di conoscenza;
- il multiforme concetto di *quantità*.

A

TREATISE

OF

ALGEBRA,

IN

THREE PARTS.

CONTAINING

I. THE FUNDAMENTAL RULES AND OPERATIONS.

II. THE COMPOSITION AND RESOLUTION OF EQUATIONS OF ALL DEGREES; AND THE DIFFERENT AFFECTIONS OF THEIR ROOTS.

III. THE APPLICATION OF ALGEBRA AND GEOMETRY TO EACH OTHER.

TO WHICH IS ADDED, AN

APPENDIX,

CONCERNING THE

GENERAL PROPERTIES OF GEOMETRICAL LINES.

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A TREATISE OF ALGEBRA.

PART I.

CHAP. I.

Definitions and Illustrations.

§ 1. ALGEBRA is a general method of computation by certain signs and symbols which have been contrived for this purpose, and found convenient. It is called an UNIVERSAL ARITHMETICK, and proceeds by operations and rules similar to those in common arithmetick, founded upon the same principles. This, however, is no argument against its usefulness or evidence; since arithmetick is not to be the less valued

valued that it is common, and is allowed to be one of the moſt clear and evident of the ſciences. But as a number of ſymbols are admitted into this ſcience, being neceſſary for giving it that extent and generality which is its greateſt excellence; the import of thoſe ſymbols is to be clearly ſtated, that no obſcurity or error may ariſe from the frequent uſe and complication of them.

§ 2. In GEOMETRY, lines are repreſented by a line, triangles by a triangle, and other figures by a figure of the ſame kind; but, in ALGEBRA, quantities are repreſented by the ſame letters of the alphabet; and various ſigns have been imagined for repreſenting their affections, relations, and dependencies. In Geometry the repreſentations are more natural, in Algebra more arbitrary: the former are like the firſt attempts towards the expreſſion of objects, which was by drawing their reſemblances; the latter correſpond more to the preſent uſe of languages and writing. Thus the evidence of Geometry is ſometimes more ſimple and obvious; but the uſe of Algebra more extenſive, and often more ready: eſpecially ſince the mathematical ſciences have acquired ſo vaſt an extent, and have been applied to ſo many enquiries.

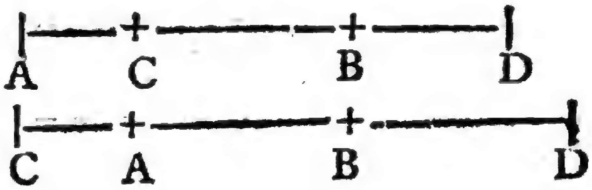
§ 3. In thoſe ſciences, it is not barely magnitude that is the object of contemplation: but there

there are many affections and properties of quantities, and operations to be performed upon them, that are neceſſarily to be conſidered. In eſtimating the ratio or proportion of quantities, magnitude only is conſidered. *(Elem.* 5. *Def.* 3.*)* But the nature and properties of figures depend on the poſition of the lines that bound them, as well as on their magnitude. In treating of motion, the direction of motion as well as its velocity; and the direction of powers that generate or deſtroy motion, as well as their forces, muſt be regarded. In OPTICS, the poſition, brightneſs, and diſtinctneſs of images, are of no leſs importance than their bigneſs; and the like is to be ſaid of other ſciences. It is neceſſary therefore that other ſymbols be admitted into Algebra beſide the letters and numbers which repreſent the magnitude of quantities.

§ 4. The relation of equality is expreſſed by the ſign $=$; thus to expreſs that the quantity repreſented by a is equal to that which is repreſented by b, we write $a = b$. But if we would expreſs that a is greater than b, we write $a \sqsubset b$; and if we would expreſs algebraically that a is leſs than b, we write $a \sqsupset b$.

§ 5. QUANTITY is what is made up of parts, or is capable of being greater or leſs. It is increaſed by *Addition*, and diminiſhed by *Subtraction*; which are therefore the two primary operations that relate to quantity. Hence it is, that any quantity may be ſuppoſed to enter into algebraic computations two different ways which have contrary effects; either as an *increment* or as a *decrement*; that is, as a quantity to be added or as a quantity to be ſubtracted. The ſign + (*plus*) is the mark of *Addition*, and the ſign — (*minus*) of *Subtraction*. Thus the quantity being repreſented by a, $+a$ imports that a is to be added, or repreſents an increment; but $-a$ imports that a is to be ſubtracted, and repreſents a decrement. When ſeveral ſuch quantities are joined, the ſigns ſerve to ſhew which are to be added, and which are to be ſubtracted. Thus $+a+b$ denotes the quantity that ariſes when a and b are both conſidered as increments, and therefore expreſſes the ſum of a and b. But $+a-b$ denotes the quantity that ariſes when from the quantity a the quantity b is ſubtracted; and expreſſes the exceſs of a above b. When a is greater than b, then $a-b$ is itſelf an increment; when $a=b$, then $a-b=0$; and when a is leſs than b, then $a-b$ is itſelf a decrement.

§ 6. As addition and ſubtraction are oppoſite, or an increment is oppoſite to a decrement, there is an analogous oppoſition between the affections of quantities that are conſidered in the mathematical ſciences. As between exceſs and defect; between the value of effects or or money due to a man, and money due by him; a line drawn towards the right, and a line drawn to the left; gravity and levity; elevation above the horizon, and depreſſion below it. When two quantities equal in reſpect of magnitude, but of thoſe oppoſite kinds, are joined together, and conceived to take place in the ſame ſubject, they deſtroy each other's effect, and their amount is *nothing*. Thus 100*l.* due to a man and 100*l.* due by him balance each other, and in eſtimating his ſtock may be both neglected. Power is ſuſtained by an equal power acting on the ſame body with a contrary direction, and neither have effect. When two unequal quantities of thoſe oppoſite qualities are joined in the ſame ſubject, the greater prevails by their difference. And when a greater quantity is taken from a leſſer of the ſame kind, the remainder becomes of the oppoſite kind. Thus if we add the lines AB and BD together, their ſum is AD; but if we are to ſubtract BD from AB,



then BC=BD is to be taken the contrary way towards A, and the remainder is AC; which, when BD, or BC exceeds AB, becomes a line on the other ſide of A. When two powers or forces are to be added together, their ſum acts upon the body: but when we are to ſubtract one of them from the other, we conceive that which is to be ſubtracted to be a power with an oppoſite direction; and if it be greater than the other, it will prevail by the difference. This change of quality however only takes place where the quantity is of ſuch a nature as to admit of ſuch a contrariety or oppoſition. We know nothing analogous to it in quantity abſtractly conſidered; and cannot ſubtract a greater quantity of matter from a leſſer, or a greater quantity of light from a leſſer. And the application of this doctrine to any art or ſcience is to be derived from the known principles of the ſcience.

§ 7. A quantity that is to be added is likewiſe called a *poſitive* quantity; and a quantity to be ſubtracted is ſaid to be *negative*: they are equally real, but oppoſite to each other, ſo as to take away each other's effect, in any operation, when they are equal as to quantity. Thus $3 - 3 = 0$, and $a - a = 0$. But though $+a$ and $-a$ are equal as to quantity, we do not ſuppoſe in Algebra that $+a = -a$; becauſe to infer equality in this ſcience, they muſt not only be equal as to quantity, but of the ſame quality, that in every operation the one may have the ſame effect as the other. A decrement may be equal to an increment, but it has in all operations a contrary effect; a motion downwards

wards may be equal to a motion upwards, and the depreſſion of a ſtar below the horizon may be equal to the elevation of a ſtar above it; but thoſe poſitions are oppoſite, and the diſtance of the ſtars is greater than if one of them was at the horizon ſo as to have no elevation above it, or depreſſion below it. It is on account of this contrariety that a negative quantity is ſaid to be leſs than nothing, becauſe it is oppoſite to the poſitive, and diminiſhes it when joined to it, whereas the addition of o has no effect. But a negative is to be conſidered no leſs as a real quantity than the poſitive. Quantities that have no ſign prefixed to them are underſtood to be poſitive.

§ 8. The number prefixed to a letter is called the numeral *coefficient*, and ſhews how often the quantity repreſented by the letter is to be taken. Thus $2a$ imports that the quantity repreſented by a is to be taken twice; $3a$ that it is to be taken thrice; and ſo on. When no number is prefixed, *unit* is underſtood to be the coefficient. Thus 1 is the coefficient of a or of b.

Quantities are ſaid to be *like* or *ſimilar*, that are repreſented by the ſame letter or letters equally repeated. Thus $+ 3a$ and $- 5a$ are like; but a and b, or a and aa are unlike.

A quantity is ſaid to conſiſt of as many *terms* as there are parts joined by the ſigns +

or —; thus $a + b$ consists of two terms, and is called a *binomial*; $a + b + c$ consists of three terms, and is called a *trinomial*. These are called *compound* quantities: a *simple* quantity consists of one term only, as $+ a$, or $+ ab$, or $+ abc$.

The other symbols and definitions necessary in *Algebra* shall be explained in their proper places.