Storia e Fondamenti della Matematica
a.a. 2020/2021

Traccia d'esame – Luglio 2021 - 2

Le radici dell'algebra affondano, in parte, nella tradizione aritmetica e geometrica di stampo euclideo. Alla luce del presente brano, tratto da una versione inglese dell'opera *De numeris datis* di Giordano Nemorario (XIII secolo), si trattino i seguenti aspetti, anche con riferimento ad altre fonti di questo o di altri autori, secondo una prospettiva storica:

- il *dato* nella matematica greca antica;
- la struttura del testo di problemi e teoremi;
- il ruolo dell'esempio numerico nella conoscenza matematica;
- il concetto di *analisi* nei procedimenti risolutivi.

Introduction

He followed not the synthetic but
the analytic way of teaching.
MARINUS on Euclid's *Data*

The *De numeris datis* of Jordanus de Nemore is recognized as the first advanced algebra composed in western Europe. The text assumes the reader's familiarity with fundamental algebraic concepts and skills, and offers a development of quadratic, simultaneous, and proportional equations, for the most part previously unexpressed. Works were available at the beginning of the late twelfth century to provide the foundation. Through them the student became familiar with equations both simple and quadratic, rules for multiplying (what we call) positive and negative integers, monomials and binomials, extraction of roots, and the use of parameters, false position, and the rule of three. Expertise in the fundamentals of algebra signaled a need for a development that would be more abstract and profound. Jordanus provided it.

Book One

Definitions.

1. *A number is given* whose quantity is known;
2. *A number is given in relation to another* where the ratio of the one to the other is given;
3. *A ratio is given* whose denomination[1] is known.

I-1. IF A GIVEN NUMBER IS SEPARATED INTO TWO PARTS WHOSE DIFFERENCE IS KNOWN, THEN EACH OF THE PARTS CAN BE FOUND.
Since the lesser part and the difference equal the larger, the lesser with another equal to itself together with the difference make the given number. Subtracting therefore the difference from the total, what remains is twice the lesser. Halving this yields the smaller and, consequently, the greater part.

For example, separate 10 into two parts whose difference is 2. If that is subtracted from 10, 8 remains, whose half is 4. This is the smaller number and the other is 6.

I-2. IF A GIVEN NUMBER IS SEPARATED INTO AS MANY PARTS AS DESIRED WHOSE SUCCESSIVE DIFFERENCES ARE KNOWN, THEN EACH OF THE PARTS CAN BE FOUND.
Given is the number a which is divided into w, x, y, and z the least of the parts. Since the successive differences of all these are given, each difference can be expressed in terms of the difference of each number with z. Therefore let f be the difference of w and z, and the sum of g and h be the sum of the differences of x and z with y and z. Now because z makes each of

those equal to each of these, it is obvious that thrice z with the sum of f, g, and h equals those three. Therefore four times z with the sum of f, g, and h equals a. Subtracting this sum from a leaves four times z, which is now known; hence z is found. By addition of differences the other parts are found.

For example, separate 40 into four parts whose, successive differences are 4, 3, and 2. Therefore the difference of the first and last is 9, of the second and last is 5, and of the third and last is 2. Their sum is 16. This subtracted from 40 leaves 24 whose fourth is 6, the least of the four parts. By adding this to 9, 5, and 2, the other three parts are found, namely 8, 11, and 15.

I-3. IF A GIVEN NUMBER IS SEPARATED INTO TWO PARTS SUCH THAT THE PRODUCT OF THE PARTS IS KNOWN, THEN EACH OF THE PARTS CAN BE FOUND.[2]

Let the given number a be separated into x and y so that the product of x and y is given as b. Moreover, let the square of $x + y$ be e, and the quadruple of b be f. Subtract this from e to get g, which will then be the square of the difference of x and y. Take the square root of g and call it h. h is also the difference of x and y. Since h is known, then x and y can be found.

The mechanics of this is easily done thus. For example, separate 10 into two numbers whose product is 21. The quadruple of this is 84, which subtracted from the square of 10, namely from 100, yields 16. 4 is the root of this and also the difference of the two parts. Subtracting this from 10 to get 6, which halved yields 3, the lesser part; and the greater is 7.

I-4. IF A GIVEN NUMBER IS SEPARATED INTO TWO PARTS THE SUM OF WHOSE SQUARES IS KNOWN, THEN EACH OF THE PARTS CAN BE FOUND.[3]

As in the previous method, call the known [sum of the squares] b, and [let] e—twice the product of the two parts—[be found by subtracting the sum of the squares from the square of the given number—*trans*]. Subtracting e from b yields h, the square of the differences,[4] whose root is c. Hence, all the parts can be found.

For example, separate 10 into two parts, the sum of whose squares is 58. Subtract this from 100 to get 42, which in turn is subtracted from 58 to yield 16. The root of this is 4, which is the difference of the parts. As before[5] these are found to be 7 and 3.

I-5. IF A NUMBER IS SEPARATED INTO TWO PARTS WHOSE DIFFERENCE IS KNOWN AND WHOSE PRODUCT IS ALSO KNOWN, THEN THE NUMBER CAN BE FOUND.

As before [6] let a be the known difference of the parts and b be their given product. Let h be the square of the difference of the parts and e be four times their product. Call the sum of these f. f is equal to the square of $x + y$.[7] And so $x + y$ is found.

For example, let the difference of the parts be 6 and their product, 16. Twice this is 32, and twice again is 64. To this is added 36, the square of 6, to make 100. The root of this is 10, the number that was separated into 8 and 2.[8]

I-6. IF THE DIFFERENCE OF TWO PARTS OF A NUMBER IS KNOWN AND ALSO THE SUM OF THEIR SQUARES, THEN THE NUMBER CAN BE FOUND.

Subtract from b, the given sum of the squares, h, the square of their given difference. Call the remainder e, which is also twice the product of the parts. The sum of e and b is the square of the desired number, f. Hence, take its root to find the number.

For example, let 68 be the sum of the squares from which 36, the square of their difference, is subtracted. The remainder, 32, is twice the product of the parts. Adding 68 and 32 to get 100, take the root of this to get 10. This is the desired number whose parts are 8 and 2.[9]

I-7. IF ONLY ONE OF TWO PARTS OF A NUMBER IS KNOWN, PROVIDED THE SUM OF THE PRODUCT OF THE PARTS AND THE SQUARE OF THE UNKNOWN PART IS GIVEN, THEN THE NUMBER CAN BE FOUND.

Let the parts of the number be x and b, with b given. Also given is a, the sum of the product of the parts, and the square of x. Add z, equal to x, to $x + b$ so that the entire $x + b + z$ can be separated into $x + b$ and z. Now since $x + b$ times z equals the given a, and the difference of $x + b$ and z is the given b, then $x + b$ and z are found as are x and $x + b$.[10]

For example, let 6 be one of the parts and 40 the sum of the product and the square. Double 40 and redouble to get 160. Add to this 36 to obtain 196 whose root is 14. From this subtract 6 and halve the remainder to yield 4. This is the unknown part that with 6 makes the desired number 10.

I-8. IF A GIVEN NUMBER IS SEPARATED INTO TWO PARTS AND THE SUM OF THE SQUARE OF THE LESSER PART AND OF THE PRODUCT OF THE GIVEN NUMBER AND THE DIFFERENCE OF THE TWO PARTS IS KNOWN, THEN EACH OF THE PARTS CAN BE FOUND.

The square of the greater part is equal to the given sum.[11] Once its square root is found, then the other part can be found.

For example, separate 10 into two parts, and let the given sum equal 64. Its root is 8, which is the greater part, and the lesser is 2.

I-9. IF A GIVEN NUMBER IS MULTIPLIED BY THE DIFFERENCE OF ITS PARTS AND IS ADDED TO THE SQUARE OF THE GREATER PART TO MAKE ANOTHER GIVEN NUMBER, THEN EACH OF THE PARTS CAN BE FOUND.

Separate the given number a into x and y so that their difference is g. Multiply $x + y$ by g to get d. Square the greater part x to yield e. Thus the sum $d + e$ is given. Now let the square of $x + y$ be f. Thus the whole $d + e + f$ is known. Since $x + y + g$ is twice x, $d + f$ will be twice the product of $x + y$ and x. Likewise $d + e + f$ will be the sum of x squared and of twice x times $x + y$. Since $d + e + f$ and twice $x + y$ are known, so is x found together with y.[12]

For example, multiply 10 by the difference of its parts and add it to the square of the greater to make 56. Add this to 100 to make 156, which doubled then redoubled is 624. Add to this the square of 20, which is twice 10, to reach 1024. The root of this is 32, from which 20 is subtracted to yield 12. Half of this is 6, the greater part of 10, and the lesser number is 4.

I-10. IF THE SUM OF THE SQUARES OF THE PARTS OF A GIVEN NUMBER IS ADDED TO THE PRODUCT OF THAT NUMBER AND THE DIFFERENCE OF ITS PARTS TO FORM ANOTHER GIVEN NUMBER, THEN EACH OF THE PARTS CAN BE FOUND.

The given sum can be reduced to twice the square of the larger part. Hence, halve it and take its root to find the larger part.

For example, squaring and adding the two parts of 10 to the product of 10 and the difference of its parts yields 98 whose half is 49. The root of this is 7 for the larger part and the smaller is 3.

I-11. IF THE SUM IS KNOWN OF THE PRODUCT OF A GIVEN NUMBER AND THE DIFFERENCE OF ITS PARTS AND OF THE PRODUCT OF ITS PARTS, THEN EACH OF THE PARTS CAN BE FOUND.

Now the given number is also equal to the difference of its parts added to twice the smaller part. Hence its square is the sum of itself times the difference of the parts and twice the product of the lesser and of the two parts. Now the product of the lesser and the whole equals the sum of the lesser times the greater and of the square of the lesser. If, therefore, from the square of the whole is taken the sum of the product of the whole and the difference and of the product of the two parts, what remains is the sum of the square of the lesser and of the lesser times the given whole. Hence, from what has gone before [13] the lesser and the greater can be found.

For example, the sum of the product of 10 and the difference of its two parts and of the product of the two parts is 89. Subtracting this from 100 leaves 11. Doubling and redoubling this yields 44, which with 100 makes 144 whose root is 12. The difference of this and 10 is 2 whose half is 1. This is the smaller number; the greater is 9.