

Storia e Fondamenti della Matematica
a.a. 2018/2019

Traccia d'esame – Giugno 2019

Grandezza, numero, simbolo: sono molteplici gli oggetti delle operazioni matematiche. Prendendo spunto dallo scritto di George Boole, illustrare:

- le loro differenze e reciproche relazioni nell'evoluzione della matematica;
- il loro ruolo nella storia della geometria e dell'algebra;
- i possibili significati del termine *analisi* nel brano proposto;
- un esempio di fonte originale (di qualsiasi epoca) che dimostri come l'interpretazione del contenuto matematico richieda una approfondita conoscenza della lingua.

THE MATHEMATICAL ANALYSIS
OF LOGIC,

BEING AN ESSAY TOWARDS A CALCULUS
OF DEDUCTIVE REASONING.

BY GEORGE BOOLE.

Ἐπικοινωνοῦσι δὲ πᾶσαι αἱ ἐπιστῆμαι ἀλλήλαις κατὰ τὰ κοινά. Κοινὰ δὲ λέγω, οἵς χρῶνται ὡς ἐκ τούτων ἀποδεικνύντες· ἀλλ' οὐ περὶ ὧν δεικνύουσιν, οὐδὲ ὅ δεικνύουσι.

ARISTOTLE, *Anal. Post.*, lib. I. cap. XI.

CAMBRIDGE:
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PREFACE.

IN presenting this Work to public notice, I deem it not irrelevant to observe, that speculations similar to those which it records have, at different periods, occupied my thoughts. In the spring of the present year my attention was directed to the question then moved between Sir W. Hamilton and Professor De Morgan; and I was induced by the interest which it inspired, to resume the almost-forgotten thread of former inquiries. It appeared to me that, although Logic might be viewed with reference to the idea of quantity,* it had also another and a deeper system of relations. If it was lawful to regard it from *without*, as connecting itself through the medium of Number with the intuitions of Space and Time, it was lawful also to regard it from *within*, as based upon facts of another order which have their abode in the constitution of the Mind. The results of this view, and of the inquiries which it suggested, are embodied in the following Treatise.

It is not generally permitted to an Author to prescribe the mode in which his production shall be judged; but there are two conditions which I may venture to require of those who shall undertake to estimate the merits of this performance. The first is, that no preconceived notion of the impossibility of its objects shall be permitted to interfere with that candour and impartiality which the investigation of Truth demands; the second is, that their judgment of the system as a whole shall not be founded either upon the examination of only a part of it, or upon the measure of its conformity with any received system, considered as a standard of reference from which appeal is denied. It is in the general theorems which occupy the latter chapters of this work,—results to which there is no existing counterpart,—that the claims of the method, as a Calculus of Deductive Reasoning, are most fully set forth.

What may be the final estimate of the value of the system, I have neither the wish nor the right to anticipate. The estimation of a theory is

*See p. 43.

not simply determined by its truth. It also depends upon the importance of its subject, and the extent of its applications; beyond which something must still be left to the arbitrariness of human Opinion. If the utility of the application of Mathematical forms to the science of Logic were solely a question of Notation, I should be content to rest the defence of this attempt upon a principle which has been stated by an able living writer: "Whenever the nature of the subject permits the reasoning process to be without danger carried on mechanically, the language should be constructed on as mechanical principles as possible; while in the contrary case it should be so constructed, that there shall be the greatest possible obstacle to a mere mechanical use of it."^{*} In one respect, the science of Logic differs from all others; the perfection of its method is chiefly valuable as an evidence of the speculative truth of its principles. To supersede the employment of common reason, or to subject it to the rigour of technical forms, would be the last desire of one who knows the value of that intellectual toil and warfare which imparts to the mind an athletic vigour, and teaches it to contend with difficulties and to rely upon itself in emergencies.

LINCOLN, Oct. 29, 1847.

*Mill's *System of Logic, Ratiocinative and Inductive*, Vol. II. p. 292.

MATHEMATICAL ANALYSIS OF LOGIC.

INTRODUCTION.

THEY who are acquainted with the present state of the theory of Symbolical Algebra, are aware, that the validity of the processes of analysis does not depend upon the interpretation of the symbols which are employed, but solely upon the laws of their combination. Every system of interpretation which does not affect the truth of the relations supposed, is equally admissible, and it is thus that the same process may, under one scheme of interpretation, represent the solution of a question on the properties of numbers, under another, that of a geometrical problem, and under a third, that of a problem of dynamics or optics. This principle is indeed of fundamental importance; and it may with safety be affirmed, that the recent advances of pure analysis have been much assisted by the influence which it has exerted in directing the current of investigation.

But the full recognition of the consequences of this important doctrine has been, in some measure, retarded by accidental circumstances. It has happened in every known form of analysis, that the elements to be determined have been conceived as measurable by comparison with some fixed standard. The predominant idea has been that of magnitude, or more strictly, of numerical ratio. The expression of magnitude, or of operations upon magnitude, has been the express object for which the symbols of Analysis have been invented, and for which their laws have been investigated. Thus the abstractions of the modern Analysis, not less than the ostensive diagrams of the ancient Geometry, have encouraged the notion, that Mathematics are essentially, as well as actually, the Science of Magnitude.

The consideration of that view which has already been stated, as embodying the true principle of the Algebra of Symbols, would, however, lead us to infer that this conclusion is by no means necessary. If every exist-

ing interpretation is shewn to involve the idea of magnitude, it is only by induction that we can assert that no other interpretation is possible. And it may be doubted whether our experience is sufficient to render such an induction legitimate. The history of pure Analysis is, it may be said, too recent to permit us to set limits to the extent of its applications. Should we grant to the inference a high degree of probability, we might still, and with reason, maintain the sufficiency of the definition to which the principle already stated would lead us. We might justly assign it as the definitive character of a true Calculus, that it is a method resting upon the employment of Symbols, whose laws of combination are known and general, and whose results admit of a consistent interpretation. That to the existing forms of Analysis a quantitative interpretation is assigned, is the result of the circumstances by which those forms were determined, and is not to be construed into a universal condition of Analysis. It is upon the foundation of this general principle, that I purpose to establish the Calculus of Logic, and that I claim for it a place among the acknowledged forms of Mathematical Analysis, regardless that in its object and in its instruments it must at present stand alone.

That which renders Logic possible, is the existence in our minds of general notions,—our ability to conceive of a class, and to designate its individual members by a common name. The theory of Logic is thus intimately connected with that of Language. A successful attempt to express logical propositions by symbols, the laws of whose combinations should be founded upon the laws of the mental processes which they represent, would, so far, be a step toward a philosophical language. But this is a view which we need not here follow into detail.* Assuming the notion of a class,

*This view is well expressed in one of Blanco White's Letters:—"Logic is for the most part a collection of technical rules founded on classification. The Syllogism is nothing but a result of the classification of things, which the mind naturally and necessarily forms, in forming a language. All abstract terms are classifications; or rather the labels of the classes which the mind has settled."—*Memoirs of the Rev. Joseph Blanco White*, vol. II. p. 163. See also, for a very lucid introduction, Dr. Latham's *First Outlines of Logic applied to Language*, Becker's *German Grammar*, &c. Extreme Nominalists make

we are able, from any conceivable collection of objects, to separate by a mental act, those which belong to the given class, and to contemplate them apart from the rest. Such, or a similar act of election, we may conceive to be repeated. The group of individuals left under consideration may be still further limited, by mentally selecting those among them which belong to some other recognised class, as well as to the one before contemplated. And this process may be repeated with other elements of distinction, until we arrive at an individual possessing all the distinctive characters which we have taken into account, and a member, at the same time, of every class which we have enumerated. It is in fact a method similar to this which we employ whenever, in common language, we accumulate descriptive epithets for the sake of more precise definition.

Now the several mental operations which in the above case we have supposed to be performed, are subject to peculiar laws. It is possible to assign relations among them, whether as respects the repetition of a given operation or the succession of different ones, or some other particular, which are never violated. It is, for example, true that the result of two successive acts is unaffected by the order in which they are performed; and there are at least two other laws which will be pointed out in the proper place. These will perhaps to some appear so obvious as to be ranked among necessary truths, and so little important as to be undeserving of special notice. And probably they are noticed for the first time in this Essay. Yet it may with confidence be asserted, that if they were other than they are, the entire mechanism of reasoning, nay the very laws and constitution of the human intellect, would be vitally changed. A Logic might indeed exist, but it would no longer be the Logic we possess.]

Such are the elementary laws upon the existence of which, and upon their capability of exact symbolical expression, the method of the following Essay is founded; and it is presumed that the object which it seeks to attain will be thought to have been very fully accomplished. Every log-

Logic entirely dependent upon language. For the opposite view, see Cudworth's *Eternal and Immutable Morality*, Book iv. Chap. III.