SYSTEM

OF

POSITIVE POLITY

BY

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THIRD VOLUME

CONTAINING

SOCIAL DYNAMICS, OR THE GENERAL THEORY OF HUMAN PROGRESS

LONDON
LONGMANS, GREEN, AND CO.
1876

properties principally derived from the first three numbers, which I have called *sacred* (p. 107), and which also deserve to be called *spontaneous* owing to their independence of signs, and *universal* as being alone common to all the higher animals. Men's notions concerning these properties had always been sound, both when fetichist instinct first unveiled, and when theocratic wisdom further developed them. But Greek subtlety often impaired their theoretic character by endeavouring to subordinate Laws to Causes; a real retrogression, since Cause never serves any other end than to prepare the way for Law (p. 106). Nevertheless, the leading thinkers used these numerical relations chiefly as aids to a better conception of the universal harmony, deriving from them a sense of connection between all phenomena whatsoever. Not only were those synthetic views truer than our analytic prejudices would suppose, but they moreover favoured the precision necessary to Positive doctrine by establishing immutability in questions of quantity as well as in those of quality, although this ultimate extension is but rarely to be grasped (i. 377).

Rise of Linear Geometry. The two theorems of Thales.

But be the philosophic value of those arithmetical inspirations what it may, the special characteristic of Greek science was the decisive progress made in abstract geometry, the expansion of which, while it was cultivated by Priests, had been too much repressed by its practical destination. Till then all geometrical notions had dealt with the measurement of areas or of volumes, while linear geometry could not be said to exist. Yet the latter had to grow out of the other two as their natural complement, before forming as we now see the rational foundation on which they rest. For as the theory of areas and volumes always reduces comparisons between them to comparisons of lines, it became in many cases inadequate and even inapplicable so long as linear relations were unknown, while these latter again could not be understood without a previous knowledge of rectilinear figures, which are obviously reducible to triangles. The study of lines, then, was to be the earliest advance achieved by Greek genius, thanks to the twofold discovery of the great Thales, whose name will ever be associated with this first step in geometrical abstraction. His two laws, of the constancy of the sum of the angles of a rectilinear triangle, and of the proportionality of the sides in equi-angular triangles, must be regarded as closely connected. From these resulted the theory of polygons, and even the outline of the theory of the circle, as a result of the measurement of angles. There is no reason why the second law might not have been derived in a direct way from the comparison of areas, as the classical proof of it soon showed. But in point of fact it was originally a consequence of the first law, which would immediately explain the properties of parallel lines, up to that time known only in a purely inductive manner.

How he was led to them.

The importance of this fundamental advance induces me here to point out its probable connection with the general body of previously existent notions, by showing that the method of areas, employed in a somewhat exceptional way, is sufficient to lead easily to the chief theorem of Thales. That theorem substantially is, that any given angle is equal to the sum of the angles formed by any transversal with one of the sides of the given angle and the other side produced. Now, measuring each angle by the infinite area it includes, this relation is evident, if we substitute for the first of the two latter angles its opposite, and if, in the case of the second, we neglect the area of the triangle. This last process logically offers the advantage of introducing from the very outset of abstract geometry the essential principle of the infinitesimal method, that is to say, the power of mutual substitution of any magnitudes whose difference is infinitely small compared with the magnitudes themselves. Although the filiation I am suggesting must, for want of documentary proof, remain conjectural, it was of consequence to the philosophy of history to frame a clear idea of the special connection between Greek speculation and its Theocratic foundation.

He reformed Astronomy,

Thus at last arose the principal department of geometry, which, from its logical properties and scientific bearings, will ever predominate over the two other parts. Thenceforth it was continuously cultivated, and so the spontaneous institution of *Space* (i. 397) came to be developed systematically. The use of this conception was to enable us to consider extension in the abstract, apart from conorete cases ; but up to this time it had remained indistinct, for want of that effective application of the institution, which the study of lines demanded. But the importance of the new impulse given by geometry was chiefly shown in the renovation that Thales worked by its aid in the whole body of Astronomy, a science that will ever be indebted to him for its second mathematical constitution.

By substituting diagrams for mechanisms.

Its first, or theocratic, constitution, as I have shown in the preceding chapter (p. 184), was essentially characterised by the employment of mechanism adapted for representing those movements of the heavens which first attract man's attention. However rude such a method, which at present is undervalued, would remain, it was none the less truly scientific; for it was the result of systematic conceptions, and it gave rational previsions, though both one and the other were wanting in exactness. The Greeks regenerated this theocratic constitution of astronomy in the first place, by substituting constructions on a plane surface for constructions in relief, diagrams thenceforth replacing mechanical apparatus, though the latter were never altogether discarded. That substitution, however, only became possible after an effective study of linear systems, pursued even till a first conception had been formed of the relations, whether angular or spherical, between three straight lines which form different planes around a given point. It was in this way that Thales and his school, at that early date, constituted Greek astronomy, which throughout continued to rest on graphic methods. The employment of calculation, the last stage in the formation of astronomical science, though instituted by Hipparchus, could not occupy the first rank until modern times.

He discovered sphericity of the Earth.

But, save in regard to the diurnal motion, celestial science continued unsatisfactory during this second logical phase. The rest of its investigations remained empirical or confused. The fundamental movements of the heavens, however, became better understood as the substitution of diagrams for mechanism rendered determinations both easier and more precise. Local variations, at all events in respect of latitude, thenceforward grew clearly discernible, owing to the tolerably extended range of Greek exploration. A general comparison of those variations enabled the geometric mind even thus early to discover the double law, which proclaimed the spherical shape of the earth and the convergence of vertical lines. Although that decisive advance is commonly attributed to the school of Alexandria, I do not hesitate to place it as far back as the time of Thales. The reaction which it exercised on general thought must have produced in this father of philosophy the first awakening of systematic Relativity, the principle of which was established permanently in reference to such phenomena as were universally known. Nevertheless the influence, mainly positive but also negative, of that change could not fully tell until complemented by the Alexandrian theory of the earth's annual motion, which will be considered further on.

Geometry one of the roots of Algebra.

In concluding this survey of the first rise of abstract science, I must note as a main point the extension of Algebra. I have already shown in the last chapter (p. 147) how much more emphatically it long bore the mark of its geometrical than of its arithmetical parentage, though the latter rightly preponderated in the end. The real character of algebraic logic first came out in the discovery made by the theocrats of the precise relations between rectilinear areas. But it would be more especially developed when, from the very outset of Greek speculation, abstract geometry caused the study of lines to prevail. Of the two conditions peculiar to algebraic magnitudes, their indeterminateness in value has a more important influence in constituting their true generality than their being abstract in kind has. Now this indeterminateness is spontaneous in geometry, where the intervention of numbers is simply an artifice for measuring results. Prolonged efforts are needed, on the contrary, in order adequately to generalise arithmetical magnitudes; for when so generalised they seem destitute of all support, although this higher degree of abstraction is better adapted to the higher developements of mathematical reasoning, the deductions of which it simplifies.

The two laws of angles and lines discovered by the founder of Greek science spontaneously concurred in developing in a decisive manner the geometrical basis from which algebra arose. For the first theorem of Thales in a direct manner establishes an equation properly so called, while the second institutes a proportion. It is true the theocratic law of the three squares had already furnished the geometrical type of equations. But by discovering an analogous, simpler, equally efficient, and more usual relation between angles, the human mind would better generalise the equational method of reasoning, which thenceforth became susceptible of application to all magnitudes whatever. Yet despite its natural superiority, that first form of abstract deduction could not find a decisive field of exercise in geometry, until the theorem of Hipparchus had founded trigonometry (p. 271). It was more especially the second mode, then, albeit less simple and more restricted, which proved essential to the logical developement of ancient geometry. The ancient geometers, in fact, managed to develope the Theory of Proportion for the furtherance of their theoretic deductions far more than we now imagine.

Pythagoras not to be reckoned among the founders of Mathematical science.

Such was the first rise of abstract science effected by the Greeks, and essentially instituted by Thales and his school. I am compelled to exclude Pythagoras and his disciples from a share in it, profoundly as I venerate that eminent philosopher. His true influence I shall soon have occasion to consider in directly judging of the general reaction of scientific culture on the universal synthesis. But such an inquiry will be more opportune when I am in a position to review that culture as a whole, by taking into account the impulse derived from Archimedes and Hipparchus, as well as that due to Thales. Though Pythagoras may have discovered the law of the three squares for himself, theocratic thinkers had long before his time arrived at it by the direct comparison of areas.

The Law of the Three Squares was discovered by Theocrats.

The only way in which he might have claimed credit in the matter would have been by re-discovering the law deductively as a linear relation from the second theorem of Thales. Now we have no indication that he actually accomplished that deduction, though it did not transcend his logical power. The supposition that the Brahmins received this great law from him is as theoretically absurd as it is historically inaccurate. In vain is it made to rest on the analogy drawn from the communication made by Thales to the Egyptian theocrats, who before he visited them knew not how to determine the height of their pyramids by the length of the shadows cast. This resemblance is purely superficial, since it overlooks the distinction between the geometry of areas, the only form cultivated by the ancient priesthood, and that of lines, which it was reserved for Greek genius to evolve.

For the sake of the intellectual glory and even the moral honour of the noble sacerdotal castes, it was incumbent on me to make this historical explanation. Devoted as they were to their social mission, and undisturbed by theoretical doubts, they were unable to develope adequately the speculative capacity of which otherwise their synthetic genius would have admitted. Standing alone, as I do, at the point of view from which they can be judged, it behoves me scrupulously to guard the integrity