

**The commutator subgroup of the alternating group  $A_4$  is the Klein group  $V_4$ .** This is normal in  $A_4$  because it is so in  $S_4$ . Moreover, on the one hand, the quotient  $A_4/V_4$  is commutative, because it is order 3, on the other hand, if  $H$  is a proper subgroup of  $A_4$ , then

- if  $H$  is trivial,  $A_4/H \simeq A_4$  is not commutative;
- if  $H$  is non trivial, then it is not a normal subgroup: in this case  $H$  has order 2, its elements are  $id$  and, with loss of generality, the permutation  $(1, 2)(3, 4)$ , whereas its conjugate

$$(1, 2, 3)(1, 2)(3, 4)(1, 3, 2) = (1, 4)(2, 3)$$

does not belong to  $H$ .