PRECORSO

venerdì 18 settembre 2026 09:07

Def: Siano $a \in \mathbb{R}$, $a > 0$, $a \neq 1$ e $y \in \mathbb{R}$, $y > 0$.
Definiamo LOGARITMO IN BASE a DI y l'unico $x \in \mathbb{R}$ t.c. $a^x = y$

PROPRIETÀ DEI LOGARITMI:

Sia $a \in \mathbb{R}$, $a > 0$, $a \neq 1$.

1) $\forall x \in \mathbb{R}$: $\log_a a^x = x$
2) $\forall x \in \mathbb{R}, x > 0$: $a^{\log_a x} = x$.
3) $\log_a 1 = 0$ e $\log_a a = 1$
4) $\forall x, y \in \mathbb{R}$, $x, y > 0$:
   $$\log_e (x \cdot y) = \log_a x + \log_a y$$
   $$\log_e \left(\frac{x}{y}\right) = \log_a x - \log_a y$$
5) $\forall x \in \mathbb{R}$, $x > 0$: $\log_e \frac{1}{x} = -\log_a x$
6) $\forall x, y \in \mathbb{R}$, $x > 0$: $\log_e x^y = y \log_a x$
7) $\forall x > 0$: $\log_{\frac{1}{a}} x = -\log_a x$
8) $\forall x \in \mathbb{R}$, $x > 0$, $x \neq 1$:
   $$\log_a x = \frac{1}{\log_x a}$$
9) $\forall x \in \mathbb{R}, x > 0$ e
   $\forall b \in \mathbb{R}$, $b > 0$, e $b \neq 1$.
   $$\log_a x = \frac{\log_b x}{\log_b a}$$

ESEMPI

- $\log_{\frac{1}{3}} \sqrt[3]{9} = \log_{\frac{1}{3}} 9^{\frac{1}{3}} = \frac{1}{3} \log_{\frac{1}{3}} 9$
  $= -\frac{1}{3} \log_3 9 = -\frac{2}{3}$

- $\log_4 40 = \log_4(4 \cdot 10) = \log_4 4 + \log_4 10$
$$= 1 + \log_4 10.$$
$$= 1 + \log_4 2 + \log_4 5$$
$$= 1 + \frac{1}{2} + \log_4 5$$
$$= \frac{3}{2} + \log_4 5$$

- $\log_2(2^4 + 2^5) = \log_2(2^4(1+2))$
$$= \log_2 2^4 + \log_2 3$$
$$= 4 + \log_2 3$$

- $\log_3(2^{10} \cdot 3^{14}) = 10 \log_3 2 + 14$

Attenzione

$$\log_a x^4 \neq \log_a^4 x$$

$\log_a x^4 = \log_a(x^4) = 4 \log_a x$

$\log_a^4 x = (\log_a x)^4$

- $2^{\log_2 x^{\log_x 4}} = 2^{\log_2(x^{\log_x 4})} = 2^{\log_2 4} = 4$

OSS $\forall x, y \in \mathbb{R}, \forall a \in \mathbb{R}, a > 0, a \neq 1$:

1) $x = y \iff a^x = a^y$.

2) $x = y \wedge x, y > 0 \iff \log_a x = \log_a y$.

Queste equivalenze si possono usare per risolvere equazioni con esponenziali e logaritmi

ESEMPI

1) $2 \cdot 3^x = 16$

$3^x = 8$

$\log_3 3^x = \log_3 8$

$x = \log_3 8$

$x = 3 \log_3 2$

2) $5^{x^2+x-1} + 3 = 0$

$5^{x^2+x-1} = -3$

impossibile (non esistono soluzioni reali)

3) $5^{x+1} = \frac{1}{2} \cdot 6^{3x}$

$\log_5 5^{x+1} = \log_5 \left(\frac{1}{2} \cdot 6^{3x}\right)$

$x+1 = \log_5 \frac{1}{2} + \log_5 6^{3x}$

$x+1 = \log_5 \frac{1}{2} + 3x \log_5 6$

$x(1 - 3\log_5 6) = \log_5 \frac{1}{2} - 1$

$$x = \frac{\log_5 \frac{1}{2} - 1}{1 - 3\log_5 6}$$

L'esercizio si poteva risolvere anche con logaritmi in basi diverse:

$5^{x+1} = \frac{1}{2} \cdot 6^{3x}$

$(x+1)\ln 5 = \ln \frac{1}{2} + 3x \ln 6$.

$x(\ln 5 - 3\ln 6) = \ln \frac{1}{2} - \ln 5$

$$x = \frac{\ln\frac{1}{2} - \ln 5}{\ln 5 - 3\ln 6}$$

In generale:

$$a^{p(x)} = b^{q(x)}$$

Possiamo fare il logaritmo in qualsiasi base.

$$\log_a a^{p(x)} = \log_a b^{q(x)}$$

$$p(x) = q(x)\log_a b$$

4) $e^{2x} + e^{x} - 2 = 0$

$t = e^x \qquad t^2 = e^{2x}.$

L'eq. si riscrive come:

$$t^2 + t - 2 = 0$$

$$t = \frac{-1 \pm \sqrt{1+8}}{2} = \begin{cases} 1 \\ -2 \end{cases}$$

$$t = 1 \quad \vee \quad t = -2$$

$e^x = 1 \quad \vee \quad e^x = -2$ (impossibile)

$$e^x = 1 \qquad \left(\begin{aligned} \ln e^x &= \ln 1 \\ x &= 0 \end{aligned} \right)$$

$$x = 0$$

5) $32\log_4^3 x - 6\log_4 x - 1 = 0$

c.e.: $x > 0.$

$t = \log_4 x$

$32t^3 - 6t - 1 = 0. \qquad \left(\pm 1, \pm\frac{1}{2}, \pm\frac{1}{4} \dots\right)$

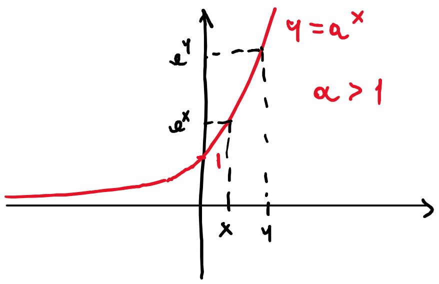
So $t = \frac{1}{2}$

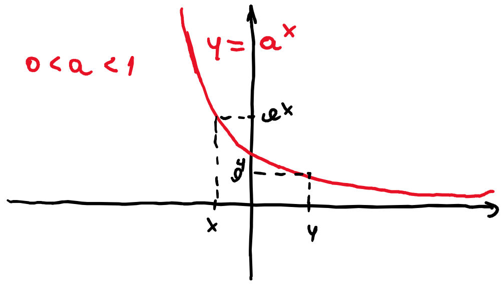
$32 \cdot \frac{1}{8} - 3 - 1 = 4 - 4 = 0.$

$$\left(t-\tfrac{1}{2}\right)\left(32t^2+16t+2\right)$$
$$2\left(t-\tfrac{1}{2}\right)\left(16t^2+8t+1\right)$$
$$2\left(t-\tfrac{1}{2}\right)\left(4t+1\right)^2=0$$
$$t=\tfrac{1}{2} \quad \vee \quad t=-\tfrac{1}{4}$$
$$\log_4 x=\tfrac{1}{2} \quad \vee \quad \log_4 x=-\tfrac{1}{4}$$
$$4^{\log_4 x}=4^{\frac{1}{2}} \qquad x=4^{-\frac{1}{4}}$$
$$x=4^{\frac{1}{2}} \qquad x=\tfrac{1}{\sqrt{2}}$$
$$x=2$$

Soluzioni: $\quad x=2 \quad \vee \quad x=\frac{1}{\sqrt{2}}$

Gli stessi metodi si usano per le disequazioni ma distinguendo le basi >1 da quelle in $]0,1[$. Il grafico della funzione esponenziale dipende dal valore della base a.





1) Se $a \in \mathbb{R}$, $a>1$. Allora:

$\forall x,y \in \mathbb{R}$:
$$x \le y \iff a^x \le a^y$$
$$x \ge y \iff a^x \ge a^y$$
$$x < y \iff a^x < a^y$$
$$x > y \iff a^x > a^y$$

Se inoltre $x,y>0$:
$$x \le y \iff \log_a x \le \log_a y$$
$$x \ge y \iff \log_a x \ge \log_a y$$
$$x < y \iff \log_a x < \log_a y$$
$$x > y \iff \log_a x > \log_a y$$

2) Se $a \in \mathbb{R}$, $0 < a < 1$ allora:

. $\forall x, y \in \mathbb{R}$:

$$x \le y \iff a^x \ge a^y.$$
$$x \ge y \iff a^x \le a^y$$
$$x < y \iff a^x > a^y$$
$$x > y \iff a^x < a^y$$

Se $x, y > 0$:

$$x \le y \iff \log_a x \ge \log_a y.$$
$$x \ge y \iff \log_a x \le \log_a y$$
$$x < y \iff \log_a x > \log_a y$$
$$x > y \iff \log_a x < \log_a y$$

ESEMPI

1) $2^{x^2-4} - 3 \le 0$

$$2^{x^2-4} \le 3$$
$$x^2 - 4 \le \log_2 3$$
$$x^2 \le 4 + \log_2 3$$
$$-\sqrt{4+\log_2 3} \le x \le \sqrt{4 + \log_2 3}$$

$x^2 \le a \quad \wedge a > 0$

$-\sqrt{a} \le x \le \sqrt{a}$

$\cancel{x \ge \pm\sqrt{a}}$

2) $\log_{\frac{1}{2}}(x+3) \ge 3.$

c.e: $x + 3 > 0 \iff x > -3$

$$\log_{\frac{1}{2}}(x+3) \ge 3$$
$$x + 3 \le \left(\frac{1}{2}\right)^3$$
$$x \le -3 + \frac{1}{8}$$
$$x \le -\frac{23}{8}$$

$$\begin{cases} x \le -\frac{23}{8} \\ x > -3 \end{cases}$$

$$-3 < x \le -\frac{23}{8}$$

Oppure:

$-\log_2(x+3) \ge 3$

$\log_2(x+3) \le -3$

$x + 3 \le 2^{-3}$

$x + 3 \le \frac{1}{8}$

.
.
.

3) $$\frac{1}{\log_5(x-1)} \le -1$$

c.e: • $x > 1$

• $\log_5(x-1) \neq 0$

$x - 1 \neq 5^0$

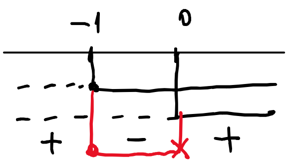
$x \neq 2$.

$t = \log_5(x-1)$

$$\frac{1}{t} \le -1$$

$$\frac{1}{t} + 1 \le 0$$

$$\frac{1+t}{t} \le 0$$



$$-1 \le t < 0$$

$$-1 \le \log_5(x-1) < 0$$

$$\frac{1}{5} \le x - 1 < 1$$

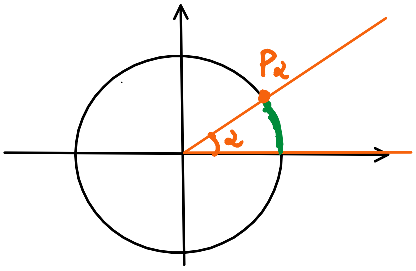
$$\frac{6}{5} \le x < 2$$

$$\begin{cases} \frac{6}{5} \le x < 2 \\ x > 1 \land x \neq 2 \end{cases} \iff \frac{6}{5} \le x < 2.$$

Cenni di Trigonometria

Chiamiamo **CIRCONFERENZA GONIOMETRICA** in la circonferenza di centro $(0,0)$ e raggio 1 (equazione: $x^2 + y^2 = 1$)

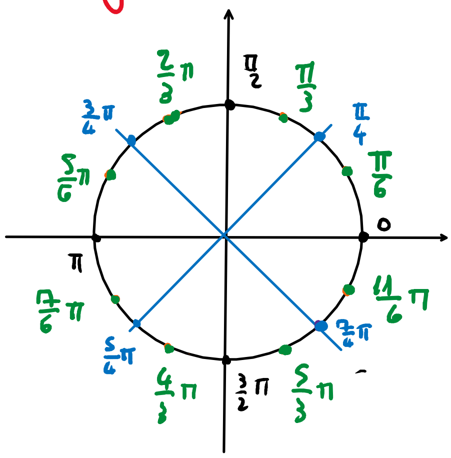
Sulla circonferenza goniometrica è facile rappresentare gli angoli: basta tracciare una semiretta e considerare l'angolo che essa forma con il semiasse orizzontale positivo.



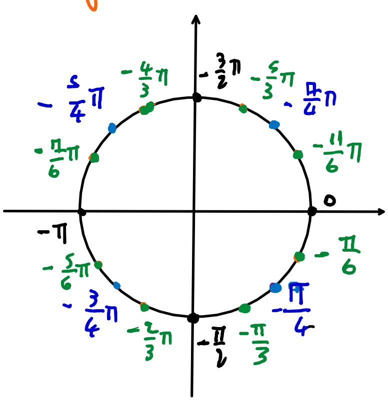
Ad ogni angolo corrisponde un arco sulla circonferenza goniometrica
La lunghezza dell'arco associato ad α si dice
MISURA IN RADIANTI DI α

Gradi	Radianti
$0°$	0
$360°$	2π
$180°$	π
$90°$	$\frac{\pi}{2}$
$45°$	$\frac{\pi}{4}$
$30°$	$\frac{\pi}{6}$
$60°$	$\frac{\pi}{3}$
$135°$	$\frac{\pi}{2} + \frac{\pi}{4} = \frac{3}{4}\pi$

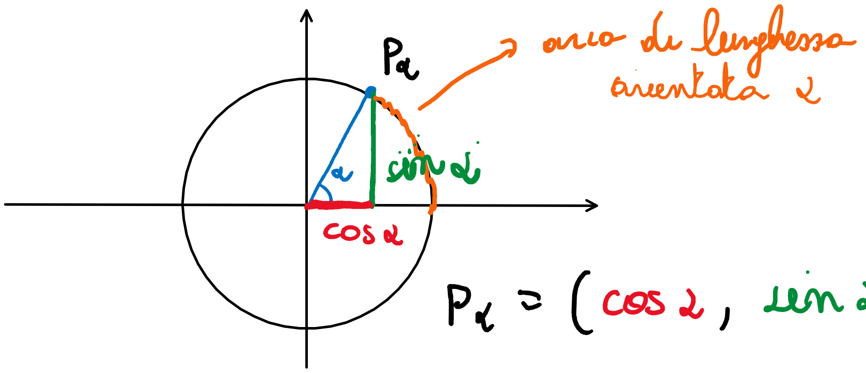
$\rightarrow$ per definizione di π circonferenza goniometrica ha lunghezza 2π)



Si possono considerare anche angoli negativi. Per convenzione questi si ottengono ruotando in senso orario:



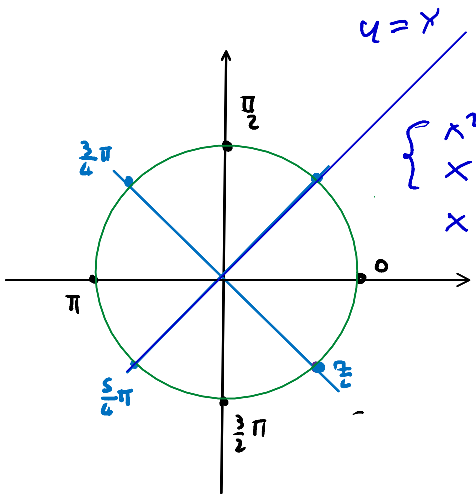
Def: Sia $\alpha \in \mathbb{R}$. Sia P_α il punto finale dell'arco sulla circ. gon. che corrisponde ad α. Le coordinate del punto P si dicono rispettivamente COSENO e SENO di α.



$$P_\alpha = (\cos \alpha, \sin \alpha)$$

Tabella di valori noti di seno e coseno

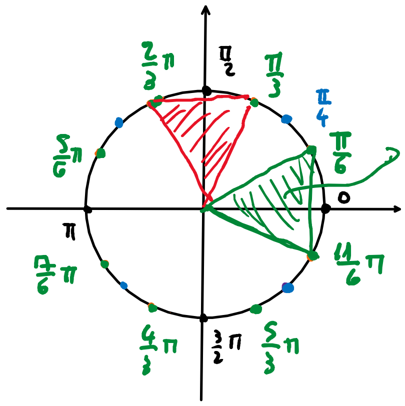
α	$\cos \alpha$	$\sin \alpha$
0	1	0
$\frac{\pi}{2}$	0	1
π	-1	0
$\frac{3}{2}\pi$	0	-1
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
$\frac{3}{4}\pi$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\frac{5}{4}\pi$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
$\frac{7}{4}\pi$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$



$$\begin{cases} x^2 + y^2 = 1 \\ x = y \end{cases}$$

$$x = y = \pm \frac{1}{\sqrt{2}}$$

$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{5}{6}\pi$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$

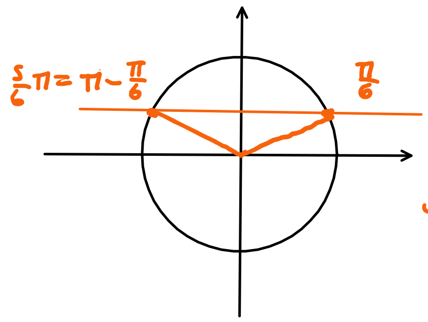


Triangolo equilatero di lato 1

$$\frac{5}{6}\pi = \pi - \frac{\pi}{6}$$

$$\cos\left(\frac{5}{6}\pi\right) = -\frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{5}{6}\pi\right) = \frac{1}{2}$$



I punti che corrispondono a $\frac{\pi}{6}$ e $\frac{5}{6}\pi$ hanno la stessa coordinata y e coordinata x opposta.

Curiosità "regola del 4":

Ordinando in maniera crescente gli angoli $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ i valori del loro coseno (risp. seno) si ottengono elencando in maniera decrescente (risp. crescente) i numeri naturali ≤ 4, dividendoli per 4 e facendone la radice quadrata:

α	$\cos\alpha$	$\sin\alpha$
0	$\sqrt{\frac{4}{4}} = 1$	$\sqrt{\frac{0}{4}} = 0$
$\frac{\pi}{6}$	$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$	$\sqrt{\frac{1}{4}} = \frac{1}{2}$
$\frac{\pi}{4}$	$\sqrt{\frac{2}{4}} = \frac{\sqrt{2}}{2}$	$\sqrt{\frac{2}{4}} = \frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\sqrt{\frac{1}{4}} = \frac{1}{2}$	$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	$\sqrt{\frac{0}{4}} = 0$	$\sqrt{\frac{4}{4}} = 1$

PROPRIETÀ

1) $\forall \alpha \in \mathbb{R}: \quad \cos^2\alpha + \sin^2\alpha = 1$

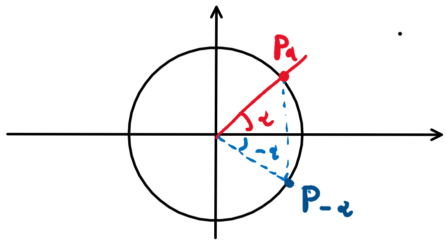
$$\left(|\cos\alpha| = \sqrt{1 - \sin^2\alpha} \right)$$

2) $\forall\, \alpha \in \mathbb{R}$: $\quad |\cos\alpha| \le 1 \quad , \quad |\sin\alpha| \le 1.$

$(-1 \le \cos\alpha \le 1 \quad , \quad -1 \le \sin\alpha \le 1)$

3) $\forall\, \alpha \in \mathbb{R}$: $\quad \cos(-\alpha) = \cos\alpha$

$\sin(-\alpha) = -\sin\alpha$



4) $\forall\, \alpha \in \mathbb{R}$ e $\forall\, k \in \mathbb{Z}$:

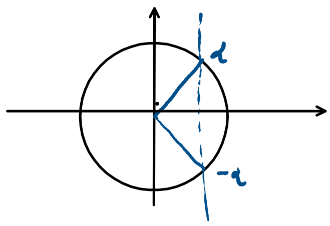
$\cos\alpha = \cos(\alpha + 2k\pi)$

$\sin\alpha = \sin(\alpha + 2k\pi).$

5) $\forall\, \alpha, \beta \in \mathbb{R}$: $\quad \cos\alpha = \cos\beta$

$\iff \alpha = \beta + 2k\pi$

$\vee \ \alpha = -\beta + 2k\pi \quad$ con $k \in \mathbb{Z}$.



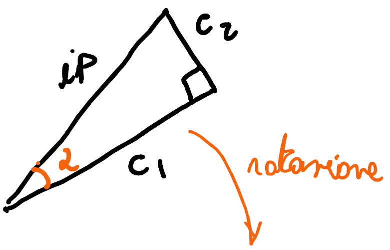
6) $\forall\, \alpha, \beta \in \mathbb{R}$: $\quad \sin\alpha = \sin\beta$

$\iff \alpha = \beta + 2k\pi$

$\vee \ \alpha = \pi - \beta + 2k\pi$

con $k \in \mathbb{Z}$.

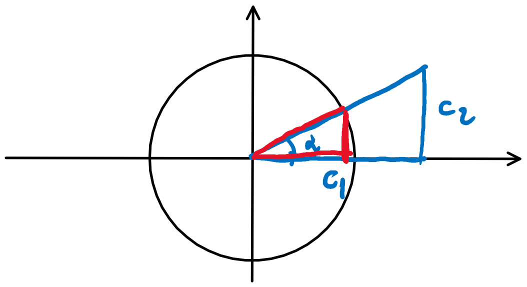
Seno e coseno nei triangoli rettangoli



ip ipotenusa

c_1 cateto adiacente ad α

c_2 cateto opposto.



$$\frac{ip}{1} = \frac{c_1}{\cos\alpha}$$

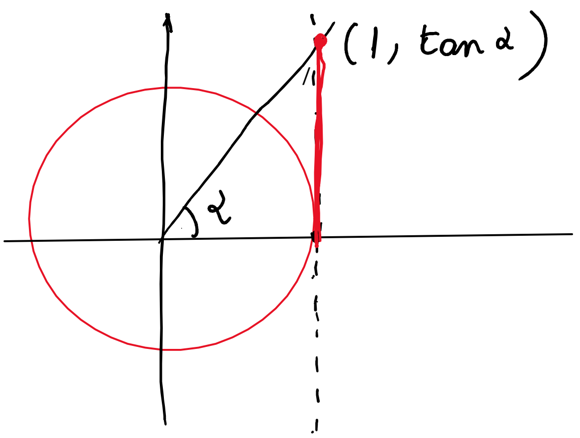
$$c_1 = ip \cdot \cos\alpha$$

In modo simile

$$c_2 = ip \cdot \sin\alpha .$$

$$\frac{c_2}{c_1} = \frac{\sin\alpha}{\cos\alpha}$$ TANGENTE DI α $(\tan\alpha \ , \ tg\,\alpha)$

$$\frac{c_1}{c_2} = \frac{\cos\alpha}{\sin\alpha}$$ COTANGENTE DI α $(\cot\alpha \ , \ cotg\,\alpha \ , \ cotan\,\alpha)$



Attenzione

1) $\tan\alpha$ è ben definito solo se $\cos\alpha \neq 0$.
cioè $\alpha \neq \frac{\pi}{2} + k\pi$ con $k \in \mathbb{Z}$.

2) $\cot\alpha$ è ben defin solo se $\sin\alpha \neq 0$ cioè
$\alpha \neq k\pi$ con $k \in \mathbb{Z}$.

$$\tan\frac{\pi}{4} = 1 \qquad \tan\frac{5}{4}\pi = 1$$

$$\tan\frac{\pi}{6} = \frac{\sqrt{3}}{3} \quad \left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}\right) \qquad \tan\frac{3}{4}\pi = -1$$

$$\tan\frac{\pi}{3} = \sqrt{3}$$

Valori di tan e cot

α	$\cos\alpha$	$\sin\alpha$	$\tan\alpha$	$\cot\alpha$
0	1	0	0	n.d
$\frac{\pi}{2}$	0	1	n.d	0
π	-1	0	0	n.d
$\frac{3}{2}\pi$	0	-1	n.d	0
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	1
$\frac{3}{4}\pi$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	-1	-1
$\frac{5}{4}\pi$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	1	1
$\frac{7}{4}\pi$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	-1	-1
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$
$\frac{2}{3}\pi$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$
$\frac{5}{6}\pi$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\frac{1}{\sqrt{3}}$	$-\sqrt{3}$
$-\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	-1	-1

Formule utili:

FORMULE DI ADDIZIONE / SOTTRAZIONE

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

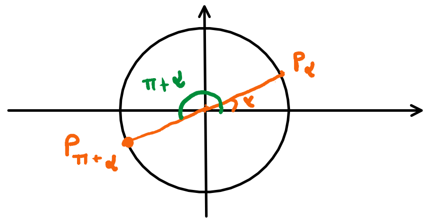
$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

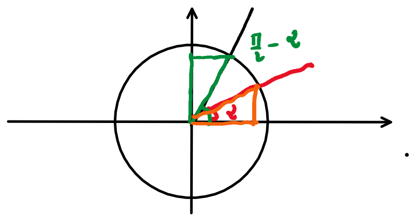
$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta.$$

Casi particolare:

$$\cos(\alpha + \pi) = -\cos\alpha$$
$$\operatorname{sen}(\alpha + \pi) = -\operatorname{sen}\alpha$$



$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$$
$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$$



$$\cos\left(\alpha - \frac{\pi}{2}\right) = \sin\alpha$$
$$\sin\left(\alpha - \frac{\pi}{2}\right) = -\cos\alpha$$

FORMULE DI DUPLICAZIONE

1) $\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha \quad \left(= 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1\right)$

2) $\sin(2\alpha) = 2\sin\alpha\cos\alpha$

FORMULE DI BISEZIONE

$$\cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2} \quad e \quad \sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2}$$

$\left(\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}} \quad e \quad \sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}} \right.$
il segno va determinato a seconda di $\left.\alpha\right)$

$$\cos\frac{\pi}{8} = \cos\left(\frac{1}{2}\frac{\pi}{4}\right) = \sqrt{\frac{1+\cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1+\frac{1}{\sqrt{2}}}{2}}$$

$$\cos\left(\frac{9}{8}\pi\right) = -\sqrt{\frac{1+\cos\left(\frac{9}{4}\pi\right)}{2}} = -\sqrt{\frac{1+\cos\left(\frac{\pi}{4}\right)}{2}} = -\sqrt{\frac{1+\frac{1}{\sqrt{2}}}{2}}$$

<u>Nota</u>: le formule di bisezione si ricavano dalle formule di duplicazione del coseno.

- $\cos 2\alpha = 2\cos^2\alpha - 1$

$\cos\alpha = 2\cos^2\frac{\alpha}{2} - 1 \Rightarrow \cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2}$

- $\cos 2\alpha = 1 - 2\sin^2\alpha$

$\cos\alpha = 1 - 2\sin^2\frac{\alpha}{2} \Rightarrow \sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2}$

FORMULE DI WERNER

$$\cos\alpha\cos\beta = \frac{1}{2}\left(\cos(\alpha+\beta) + \cos(\alpha-\beta)\right)$$

$$\sin\alpha\sin\beta = \frac{1}{2}\left(\cos(\alpha-\beta) - \cos(\alpha+\beta)\right)$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left(\sin(\alpha+\beta) + \sin(\alpha-\beta)\right)$$

$$\cos\alpha\sin\beta = \frac{1}{2}\left(\sin(\alpha+\beta) - \sin(\alpha-\beta)\right)$$

Ponendo $x = \alpha+\beta$ e $y = \alpha-\beta$, così che $\alpha = \frac{x+y}{2}$ e $\beta = \frac{x-y}{2}$ si ricavano le seguenti formule.

FORMULE DI PROSTAFERESI

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

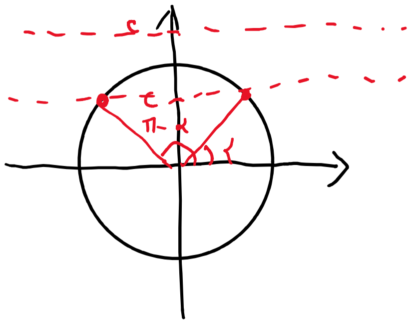
$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right)$$

Equazioni elementari:

$\sin x = c \qquad c \in \mathbb{R}$;



- Se $c > 1 \vee c < -1$ l'equazione non ha soluzioni.
- Se $c = 1$:
  $x = \frac{\pi}{2} + 2k\pi$ con $k \in \mathbb{Z}$
- Se $c = -1$:
  $x = \frac{3}{2}\pi + 2k\pi$ con $k \in \mathbb{Z}$
- Se $-1 < c < 1$
  Se conosciamo un angolo α t.c. $\sin \alpha = c$.

$$\sin x = \sin \alpha$$

$$x = \alpha + 2k\pi \quad \vee \quad x = \pi - \alpha + 2k\pi$$

OSS Spesso le soluzioni si possono scrivere in infiniti modi diversi ma equivalenti.

$x = \frac{\pi}{2} + 2k\pi$ con $k \in \mathbb{Z}$ $\iff$ $x = -3\frac{\pi}{2} + 2k\pi$ con $k \in \mathbb{Z}$

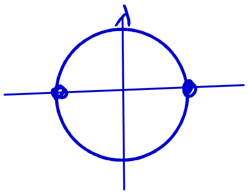
$\iff$ $x = \frac{5}{2}\pi + 2k\pi$

OSS 2

Se $c = 0$, si può prendere $\alpha = 0$.

$\sin x = 0 \iff x = 0 + 2k\pi \ \vee \ x = \pi + 2k\pi$

$\iff x = 0 + k\pi$ con $k \in \mathbb{Z}$.



ESEMPI

1) $2 \sin x = 1$

$\sin x = \frac{1}{2}$

$\sin x = \sin \frac{\pi}{6}$

$x = \frac{\pi}{6} + 2k\pi \quad \vee \quad x = \pi - \frac{\pi}{6} + 2k\pi \quad$ con $k \in \mathbb{Z}$

$= \frac{5}{6}\pi + 2k\pi \quad$ con $k \in \mathbb{Z}$.

2) $2\sin(3x) = \sqrt{3}$

$\sin(3x) = \frac{\sqrt{3}}{2}$

$\sin(3x) = \sin\left(\frac{\pi}{3}\right)$

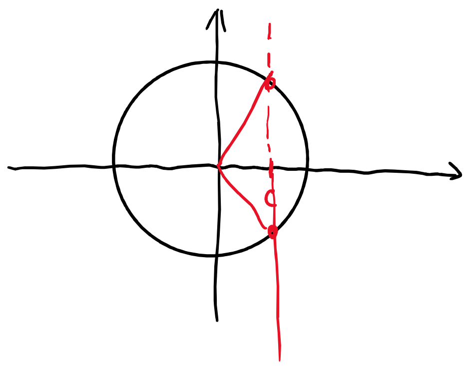
$3x = \frac{\pi}{3} + 2k\pi \quad \vee \quad 3x = \frac{2}{3}\pi + 2k\pi$

$x = \frac{\pi}{9} + \frac{2k\pi}{3} \quad \vee \quad x = \frac{2}{9}\pi + \frac{2k\pi}{3} \quad$ con $k \in \mathbb{Z}$.

$\cos x = c$.

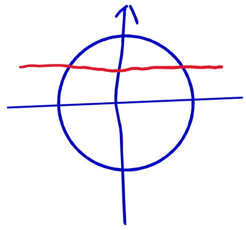
- Se $c > 1 \vee c < -1$: non ci sono soluzioni.
- Se $c = 1$: $x = 2k\pi$ con $k \in \mathbb{Z}$
- Se $c = -1$: $x = \pi + 2k\pi$ con $k \in \mathbb{Z}$
- Se $-1 < c < 1$: e conosciamo α tale che $\cos\alpha = c$, allora:

$x = \alpha + 2k\pi \quad \vee \quad x = \underbrace{-\alpha}_{\text{o } 2\pi - \alpha} + 2k\pi$, $k \in \mathbb{Z}$.



$$\cos x = \frac{\sqrt{2}}{2} \qquad \left(= \cos \frac{\pi}{4}\right)$$

$$x = \frac{\pi}{4} + 2k\pi \qquad \vee \qquad x = -\frac{\pi}{4} + 2k\pi$$



Def: Sia $c \in [-1, 1]$, si definisce ARCOSENO di c l'unico numero reale $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ tale che $\sin x = c$

(si indica con $\arcsin c$)

$$\sin x = \frac{1}{3} \qquad \left(\frac{1}{3} = \sin\left(\arcsin \frac{1}{3}\right)\right)$$

$$x = \arcsin \frac{1}{3} + 2k\pi \qquad \vee \qquad x = \pi - \arcsin \frac{1}{3} + 2k\pi$$

Def: Sia $c \in \mathbb{R}$, $c \in [-1, 1]$. Definiamo ARCOCOSENO di c come l'unico $x \in [0, \pi]$ tale che $\cos x = c$. ($\arccos c$)

$$\cos(2x) = \frac{1}{4}$$

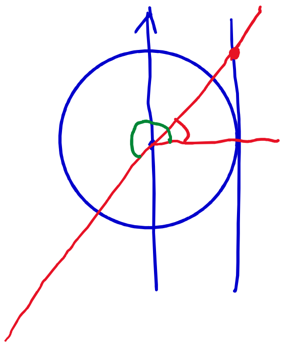
$$\cos(2x) = \cos\left(\arccos \frac{1}{4}\right)$$

$$2x = \arccos \frac{1}{4} + 2k\pi \qquad \vee \qquad 2x = -\arccos \frac{1}{4} + 2k\pi$$

$$x = \frac{1}{2} \arccos \frac{1}{4} + k\pi \qquad \vee \qquad x = -\frac{1}{2} \arccos \frac{1}{4} + k\pi.$$

OSS Siano $x, y \in \mathbb{R}$, $x, y \neq \frac{\pi}{2} + k\pi$ con $k \in \mathbb{Z}$:

$\tan x = \tan y \iff x = y + k\pi$ con $k \in \mathbb{Z}$.



$\tan x = 1$

$\tan x = \tan \frac{\pi}{4}$

$x = \frac{\pi}{4} + k\pi \quad \text{con } k \in \mathbb{Z}.$

Def: Sia $x \in \mathbb{R}$, si definisce ARCOTANGENTE di x l'unico angolo in $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$ la cui tangente è x.

$\tan x = 2$

$\tan x = \tan(\arctan 2)$

$x = \arctan 2 + k\pi \quad \text{con } k \in \mathbb{Z}.$