1.2.2 $\sqrt[3]{4-4\sqrt{5}}\;\sqrt[3]{4+4\sqrt{5}} = \sqrt[3]{(4-4\sqrt{5})(4+4\sqrt{5})} =$

$= \sqrt[3]{16-16\cdot 5} = \sqrt[3]{-64} = -\sqrt[3]{64} = -4.$

Esercizio 1.8 Dimostrare che $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\cdots}}}} \in \mathbb{N}.$

1.3.2: $\dfrac{\sqrt[3]{-8a^3b^{-2}}}{\sqrt[6]{64(-a)^2}} = \dfrac{-2ab^{-2/3}}{2\;a^{1/3}} = -\sqrt[3]{\dfrac{a^2}{b^2}}$

Esercizio 1.6 $\sqrt{7+4\sqrt{3}} - \sqrt{3} = n \quad \forall n \in \mathbb{N}.$

$\sqrt{7+4\sqrt{3}} = n+\sqrt{3} \Rightarrow 7+4\sqrt{3} = (n+\sqrt{3})^2 = n^2+2n\sqrt{3}+3$

$\Rightarrow 7+4\sqrt{3} = n^2+2n\sqrt{3}+3 \Rightarrow \underbrace{(4-2n)}_{\mathbb{Z}}\underbrace{\sqrt{3}}_{\mathbb{I}} = \underbrace{n^2-4}_{\in\mathbb{Z}}$

$4-2n = 0 \Rightarrow \underline{n=2}$

Qundi, se $\sqrt{7+4\sqrt{3}} - \sqrt{3} = n \Rightarrow \sqrt{7+4\sqrt{3}} - \sqrt{3} = 2.$

$\sqrt{7+4\sqrt{3}} - \sqrt{3} = 2 \Leftrightarrow \underbrace{\sqrt{7+4\sqrt{3}}}_{>0} = \underbrace{2+\sqrt{3}}_{>0} \overset{\square^2}{\Leftrightarrow}$ $\overset{\Leftarrow}{\sqrt{\ }}$

$\Leftrightarrow 7+4\sqrt{3} = (2+\sqrt{3})^2 = 4+4\sqrt{3}+3 = 7+4\sqrt{3}$

$\Leftrightarrow 0=0.$

$a, b \in \mathbb{R}$, $m = \frac{a+b}{2}$

$$a+b = 2m$$

$$\Downarrow$$

$$(a-b)(a+b) = 2m(a-b) \qquad (1)$$

$$\Updownarrow$$

$$a^2 - b^2 = 2ma - 2mb$$

$$\Updownarrow$$

$$a^2 - 2ma + m^2 = b^2 - 2mb + m^2$$

$$\Updownarrow$$

$$(a-m)^2 = (b-m)^2$$

$$\Downarrow \qquad (2)$$

$$|a-m| = |b-m|$$

$$\Updownarrow$$

$$a-m = b-m$$

$$\Updownarrow$$

$$a = b.$$

(*) $i = \sqrt{(-1)}$ $\quad i^2 = -1.$

$$-1 = i^2 = \sqrt{(-1)}\sqrt{(-1)} = \sqrt{(-1)(-1)} = \sqrt{1} = 1$$

$$-1 = 1.$$