8/9/2022
POMERIGGIO

Proprietà: Siano u, v, w vettori e k un numero reale

- $\langle (u+v), w \rangle = \langle u, w \rangle + \langle v, w \rangle$
- $\langle ku, w \rangle = k \langle u, w \rangle$
- $\langle u, v \rangle = \langle v, u \rangle$
- $\langle u, u \rangle \geq 0$ e $\langle u, u \rangle = 0 \iff u = 0$
- $\| u+v \| \leq \|u\| + \|v\|$

Caso particolare di $\mathbb{R}^3$

$i = (1,0,0)$, $j = (0,1,0)$, $k = (0,0,1)$

ES

$$\underbrace{2i - 3j + \sqrt{2}k}_{=} = (2, -3, \sqrt{2})$$

$$= 2(1,0,0) - 3(0,1,0) + \sqrt{2}(0,0,1)$$

$\langle i, i \rangle = 1 = \langle j, j \rangle = \langle k, k \rangle$

$\langle i, j \rangle = 0 = \langle i, k \rangle = \langle j, k \rangle$

Prodotto vettoriale

Siano u, v vettori di $\mathbb{R}^3$

$u = (x_u, y_u, z_u)$

$v = (x_v, y_v, z_v)$ definiamo il prodotto vettoriale $u \times v$ come segue

$$u \times v = (y_u z_v - y_v z_u,\ x_v z_u - x_u z_v,\ x_u y_v - y_u x_v)$$

Come fare per ricordare

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \longmapsto ad - bc$$

$$u \times v = i \begin{vmatrix} y_u & z_u \\ y_v & z_v \end{vmatrix} - j \begin{vmatrix} x_u & z_u \\ x_v & z_v \end{vmatrix} + k \begin{vmatrix} x_u & y_u \\ x_v & y_v \end{vmatrix}$$

$$= i(y_u z_v - y_v z_u) - j(x_u z_v - x_v z_u) + k(x_u y_v - x_v y_u)$$

$i \times j = (0, 0, 1) = k, \quad j \times k = i, \quad k \times i = j$

$i = (1, 0, 0)$

$j = (0, 1, 0)$

$j \times i = -k$

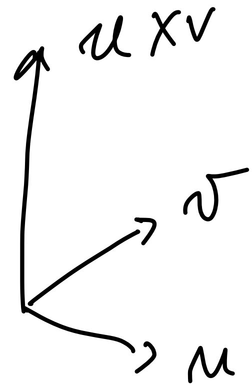
$j = (0, 1, 0)$ $\quad j \times i = (0, 0, -1) = -k$

$i = (1, 0, 0)$

- $\boxed{u \times v = -v \times u}$

ES PER CASA

$$\langle u \times v, u \rangle = \langle u \times v, v \rangle = 0$$



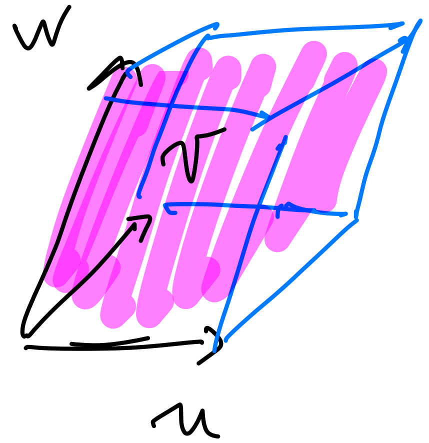
$$\|u \times v\| = \|u\| \|v\| \sin \vartheta$$

ϑ angolo compreso fra u e v

- Se u, v, w sono vettori di $\mathbb{R}^3$ allora

$$\langle u, v \times w \rangle$$

rappresenta il volume del parallelepipedo formato dai vettori u, v e w



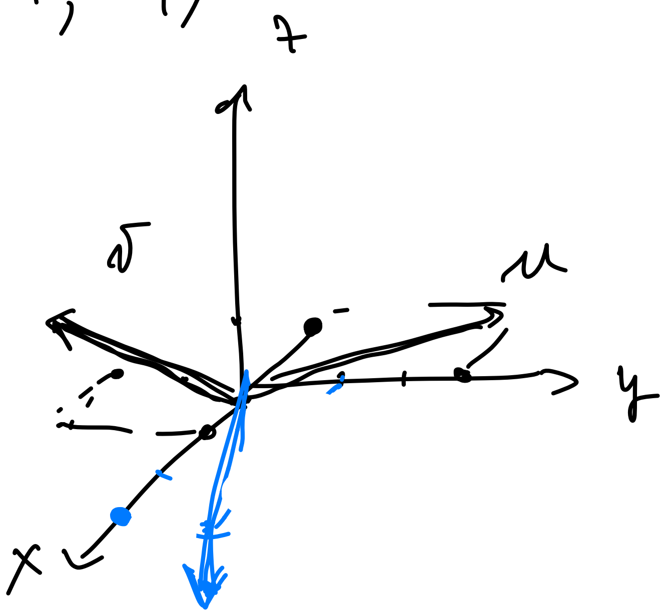
$\boxed{\text{Esempio}}$

$$u = (-1, 3, 0) \qquad v = (1, -2, 1)$$

$$u \times v = i \begin{vmatrix} 3 & 0 \\ -2 & 1 \end{vmatrix} - j \begin{vmatrix} -1 & 0 \\ 1 & 1 \end{vmatrix} + k \begin{vmatrix} -1 & 3 \\ 1 & -2 \end{vmatrix} =$$

$$= i \cdot 3 - j(-1) + k(-1) =$$

$$= (3, 1, -1)$$



Dati $v_1, v_2, \dots, v_n$ vettori e $\alpha_1, \dots, \alpha_n$ numeri reali

si dice combinazione lineare di $v_1, \dots, v_n$ a coeffic. $\alpha_1, \dots, \alpha_n$ il vettore

$$\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n$$

Esempio: Scrivere $(2,5)$ come comb. lin di $(3,-4)$ e $(2,-3)$.

Trova $\alpha_1, \alpha_2 \in \mathbb{R}$ tali che

$$(2,5) = \alpha_1(3,-4) + \alpha_2(2,-3)$$
$$= (3\alpha_1, -4\alpha_1) + (2\alpha_2, -3\alpha_2) =$$
$$= (3\alpha_1 + 2\alpha_2, -4\alpha_1 - 3\alpha_2)$$

$$\begin{cases} 2 = 3\alpha_1 + 2\alpha_2 \\ 5 = -4\alpha_1 - 3\alpha_2 \end{cases} \qquad \begin{cases} \alpha_1 = \frac{2}{3} - \frac{2}{3}\alpha_2 \\ 5 = -4\left(\frac{2}{3} - \frac{2}{3}\alpha_2\right) - 3\alpha_2 \end{cases}$$

$$\begin{cases} \text{———} \\ 5 = -\frac{8}{3} + \frac{8}{3}\alpha_2 - 3\alpha_2 \end{cases}$$

$$\begin{cases} \text{———} \\ \frac{23}{\cancel{3}} = -\frac{\alpha_2}{\cancel{3}} \end{cases} \qquad \boxed{\alpha_2 = -23}$$

$$\alpha_1 = \frac{2}{3} - \frac{2}{3}(-23)$$
$$= \frac{45}{3} = 15$$

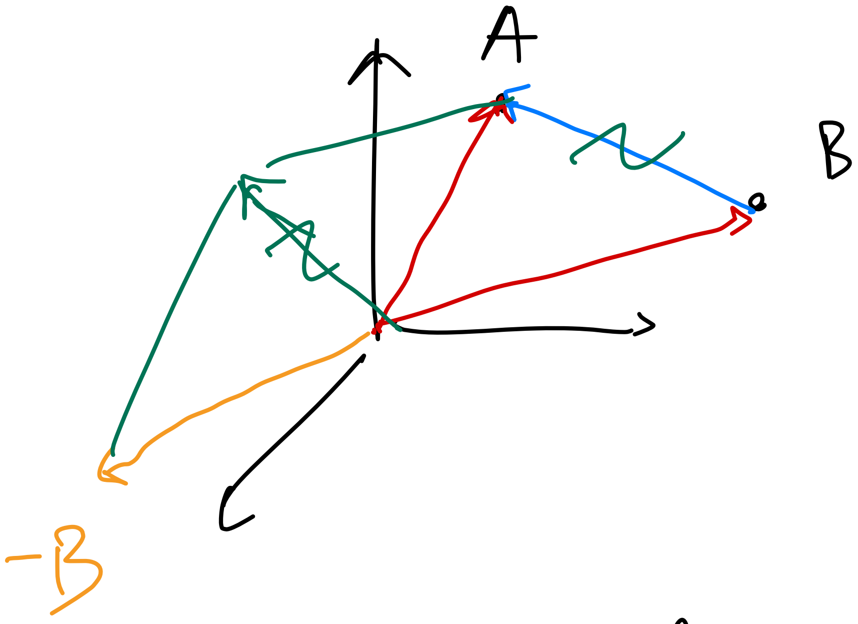
Quindi $(2,5) = 15(3,-4) - 23(2,-3)$

$$(2,5) = 5(1,2) - 1(3,5)$$

fare disegno coi quadretti.

Se identifichiamo gli elementi di $\mathbb{R}^2$ e $\mathbb{R}^3$ come punti

$$A[x_A, y_A, z_A] \qquad B[x_B, y_B, z_B]$$



Per determinare la distanza fra due punti è sufficiente calcolare la lunghezza del vettore differenza

$$d(A,B) = \| A - B \|$$

Verificare che $u = (1,2,-2)$
$v = (3,-12,4)$

$$\|u+v\| \leq \|u\| + \|v\|$$

$$u+v = (1+3,\ 2-12,\ -2+4) = (4,-10,2)$$

$$\|u\| = \sqrt{1^2+2^2+(-2)^2} = \sqrt{9} = 3$$

$$\|v\| = \sqrt{3^2+12^2+4^2} = \sqrt{169} = 13$$

$$\|u+v\| = \sqrt{16+100+4} = \sqrt{120} = \sqrt{15 \cdot 8}$$

$$= \sqrt{3 \cdot 5 \cdot 2^3} = 2\sqrt{2 \cdot 3 \cdot 5} = 2\sqrt{30}$$

$$\underbrace{2\sqrt{30}}_{5<\sqrt{30}<6} \overset{?}{\leq} 3+13 = 16$$

< 12

VERO

$$\|u+v\|^2 = \langle u+v, u+v \rangle =$$

$$= \langle u, u+v \rangle + \langle v, u+v \rangle =$$

$$= \langle u,u \rangle + \langle u,v \rangle + \langle v,u \rangle + \langle v,v \rangle$$

$$= \|u\|^2 + 2\langle u,v \rangle + \|v\|^2$$

$$\leq \|u\|^2 + 2\|u\|\|v\| + \|v\|^2 = (\ )$$

$$\left(\begin{array}{l} \dfrac{\langle u,v\rangle}{\|u\|\|v\|} = \cos\vartheta \in [-1,1] \le 1 \\ \langle u,v\rangle \le \|u\|\|v\| \end{array} \right)$$

$$= (\|u\|+\|v\|)^2$$

$\boxed{\text{ES}}$ $P(2,3,-7)$ $Q(1,-6,-5)$

$$\begin{aligned} d(P,Q) &= \|P-Q\| = \\ &= \|(1,9,-2)\| = \\ &= \sqrt{1+81+4} = \sqrt{86} \end{aligned}$$

$(2,4)$, raggio 7

$$\underbrace{d(P,(2,4)) = 7}$$

$$\| p - (2,4) \| = \| (x,y) - (2,4) \| =$$

$p = (x,y)$

$$= \| (x-2, y-4) \| =$$

$$= \sqrt{(x-2)^2 + (y-4)^2} = 7$$

$$(x-2)^2 + (y-4)^2 = 49$$