UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO

CONSIGLIO INTERCLASSE IN MATEMATICA

General information		
Academic subject	Critical Points Theorems	
Degree course	Degree in Mathematics	
Academic Year	I or II	
European Credit Transfer and Accumulation System (ECTS)		7
Language	Italian	
Academic calendar (starting and ending date)	II semester (february 2022- may 2022)	
Attendance	Non mandatory	

Professor/ Lecturer	
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Department and address	Department of Mathematics, Room 10 (fourth floor), via E. Orabona, 4 – 70125 Bari
Virtual headquarters	Microsoft Teams
Tutoring (time and day)	By appointment to be arranged upon e-mail or telephone (at the Department of Mathematics or online)

Syllabus	
Learning Objectives	Knowledge of variational and topological methods in the study of nonlinear problems, especially topological degree and index theories. Applications to the study of some semi-linear elliptic problems.
Course prerequisites	Mathematical knowledge which usually is acquired during the first three years of a degree of L-35 class. Especially: classical analysis of one and several variables, general topology, linear algebra, Hilbert spaces and L^p spaces.
Contents	**Topological degree and applications:** Topological degree for continuous functions in finite dimension: axiomatic definition and properties. Construction of the topological degree. Brower fixed point Theorem. Retraction Theorem. Borsuk Theorem. Topological linking, definition and examples: mountain pass linking, multidimensional mountain pass linking, saddle linking. Topological degree in infinite dimension. Schauder fixed point Theorem. Retraction Theorem. Other fixed point Theorems. **Index theory:** Index theory in topological spaces: axiomatic definition. Lusternik-Schnirelmann category: definition, examples and properties. Krasnoselski genus: definition, examples and properties. Relationship between the degree and the category of a set. Index theory related to a group of unitary transformations on a Hilbert space. S^1- index. **Abstract theorems of critical points existence and applications:** Generalities on the Palais-Smale condition. Deformation Lemma. Linking Theorem and applications to the study of some elliptic problems with sublinear or superlinear growth. Linking Theorem for strongly indefinite functionals. Application to the study of a first order hamiltonian system. **Abstract theorems of critical points multiplicity and applications** Deformation lemma for functionals compatible with an index theory. Abstract

	theorems of critical points multiplicity for functionals bounded from below and compatible with an index theory. Abstract theorems using the Lusternik-Schnirelmann category and applications. Study of a nonlinear eigenvalue problem. Abstract theorems of critical points multiplicity for functionals even and bounded from below. Applications to some symmetric elliptic equations. Symmetric mountain pass theorem and symmetric multidimensional mountain pass theorem. Pseudo-index theory. An abstract theorem of critical points multiplicity for functionals even and unbounded from below. Applications to some superlinear or asymptotically linear elliptic problems. Multiplicity results for strongly indefinite S^1 - invariant functionals (sketch of the proof). Application to the study of symmetric first order hamiltonian system.
Books and bibliography	P. H. Rabinowitz, *Minimax methods in critical point theory with applications to differential equations*, CBMS Regional conference Series in Applied Mathematics, 65 (1986). J.T. Scharwtz, *Nonlinear Functional Analysis*, Gordon & Breach, New York (1969). M. Struwe, *Variational methods*, Fourth edition, Springer- Berlin (2008).
Additional materials	

Work schedule			
Total	Lectures	Hands on (Laboratory, working groups, seminars, field trips)	Out-of-class study hours/ Self-study hours
Hours			
175	52	8	115
ECTS			
7	6,5	0,5	

Teaching strategy	Lessons and exercises on the various topics of the course will be held using traditional blackboard. They could be held in mixed mode, frontal and remote, or only online, if the pandemic situation requires it.
Expected learning outcomes	
Knowledge and understanding on:	Knowledge of basic and advanced tools in the study of variational problems.
Applying knowledge and understanding on:	The acquired theoretical knowledge is useful in the study of many nonlinear differential problems.
Soft skills	• *Making informed judgments and choices* Problem solving skills should be supported by the capacity in evaluating the consistency of the found solutions with the theoretical knowledge. • *Communicating knowledge and understanding* Students should acquire the mathematical language and formalism necessary to read and comprehend textbooks, to explain the acquired knowledge and to describe, analyze and solve problems. • *Capacities to continue learning* Knowledge of suitable learning methods, supported also by consultation of the texts and by solution of exercises and problems suggested during the course.

Assessment and feedback	
Methods of assessment	Oral examination which will be held using traditional blackboard, or online if the pandemic situation requires it.
Evaluation criteria	The examination consists of questions on the main topics of the course. The student must demonstrate to know variational and topological methods and to be able to apply them to the study of some semi-linear elliptic problems.
Criteria for assessment and attribution of the final mark	The final mark is expressed out of thirty. The exam is passed when the mark is greater than or equal to 18.
Additional information	