General information		
Academic subject	***Mathematical methods of physics***	
Degree course	*Mathematics*	
Academic Year	*2021/2022*	
European Credit Transfer and Accumulation System (ECTS)		7
Language	*Italian*	
Academic calendar (starting and ending date)		*First semester*
Attendance	*free*	

Professor/ Lecturer	
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Virtual headquarters	*Microsoft Teams*
Tutoring (time and day)	Online by appointment

Syllabus	
Learning Objectives	*Acquiring the basic techniques for the study of the classical equations of mathematical physics, with a particular focus on the mathematical formulation of the corresponding physical model.*
Course prerequisites	*Mathematical knowledge usually acquired during the first two years of a degree of L–35 class. Especially: classical analysis of one and several variables, electromagnetism and Hamiltonian mechanics.*

Contents	1. ***Transport equation****: Transport equation with constant coefficients. General solution and characteristic lines. Initial value problem. Weak solutions. Nonhomogeneous problem. A model of transport and Burgers equation. Introduction to scalar conservation laws. Shocks and entropy condition.* 2. ***Laplace's equation****: Outline of electrostatic and physical interpretation. Harmonic functions. Fundamental solution in R^n. Representation formula for Poisson's equation. Mean-value theorem. Maximum principle. Uniqueness. Smoothness. Estimates on derivatives. Liouville's theorem. Analyticity. Harnack's inequality. Green's functions and representation formula for Poisson's equation with boundary conditions. Symmetry of Green's function and reciprocity principle. Green's function for a half-space and a ball. Poisson's formulae. Energy methods, uniqueness, and Dirichlet's minimum principle.* 3. ***Heat equation****: Thermal conduction equation. Physical interpretation. Fundamental solution in R^n. Cauchy problem and representation formula. Nonhomogeneous problem and Duhamel's principle. Parabolic cylinder and heat ball. Mean-value theorem. Maximum principle. Uniqueness in bounded domains. Maximum principle in R^n and uniqueness of the Cauchy problem. Regularity . Estimates on derivatives. Energy methods, Forward and backward uniqueness.* 4. ***Wave equation****: Heuristic derivation and physical interpretation. Solution in 1D. D'Alembert's formula. Wave equation on the half-line. Reflection method. Spherical means and Euler-Poisson-Darboux equation. Cauchy problem in 3D. Kirchhoff's formula. Wave equation in 2D. Method of descent and Poisson's formula. Representation formulae in arbitrary even and od dimensions. Regularity. Domain of dependence and cone of influence. Huygens's principle. Nonhomogeneous problem and retarded potentials. Energy methods. Uniqueness. Finite propagation speed.* 5. ***Hamilton-Jacobi equation****: Nonlinear first-order equations. Complete integrals and envelopes. Method of characteristics. Local existence theorem. Applications. Hamilton-Jacobi equation and Hamilton's variational principle. Euler-Lagrange equations and Hamilton equations. Legendre transfom and Lagrange-Hamilton duality.*
Books and bibliography	*L.C. Evans, Partial differential equations, Graduate Studies in Mathematics, Volume 19, Amer. Math. Soc., Providence, 1998.* *F. John, Partial Differential Equations, Springer Verlag, 1982* *A.N. Tikhonov and A.A. Samarskii, Equations of Mathematical Physics, Dover Publications, 1990.*
Additional materials	

Work schedule	

Total	Lectures	Hands on (Laboratory, working groups, seminars, field trips)	Out-of-class study hours/ Self-study hours
Hours			
56	*40*	*16*	*119*
ECTS			
7			

Teaching strategy	
Expected learning outcomes	
Knowledge and understanding on:	○ Acquiring fundamental concepts and strategies for the solution of partial differential equations. ○ Acquiring basic mathematical proof techniques.
Applying knowledge and understanding on:	○ The acquired theoretical knowledge can be used in most of the differential equations of physics.
Soft skills	• *Making informed judgments and choices* ○ Ability to analyze the consistency of the logical arguments used in a proof. Problem solving skills should be supported by the capacity in evaluating the consistency of the found solution with the theoretical knowledge. • *Communicating knowledge and understanding* ○ Students should acquire the physical and mathematical language necessary to read and comprehend textbooks, to explain the acquired knowledge, and to describe, analyze and solve problems. • *Capacities to continue learning* ○ Acquiring suitable learning methods, supported by text consultation and by solving the exercises and the questions periodically assigned in class.

Assessment and feedback	
Methods of assessment	*Oral exam*
Evaluation criteria	
Criteria for assessment and attribution of the final mark	*The final mark is expressed out of thirty. The exam is passed if the final mark is greater than 17.*
Additional information	