Academic subject: Analytical Methods in Finance				
Degree Class: LM-40 – Scienze Matematiche		**Degree Course**: Mathematics	**Academic Year:** 2020/2021	
		Kind of class: optional	**Year:** 2	**Period:** 1
			ECTS: 7 divided into **ECTS lessons:** 6,5 **ECTS exe/lab/tutor:** 0,5	
Time management, hours, in-class study hours, out-of-class study hours lesson: 52 exe/lab/tutor: 8 in–class study: 60 out–of–class study: 115				
Language: Italian	**Compulsory Attendance**: no			
Subject Teacher: Mario Michele Coclite	**Tel:** +390805442659 **e–mail:** mariomichele.coclite@uniba.it	**Office:** Department of Mathematics Room 10, Floor II	**Office days and hours**: Tuesday 11–13. Other days by appointment.	

Prerequisites:
Basic elements of Lebesgue measure theory, portfolio theory, calculus of probability.

Educational objectives:
Acquiring the fundamental results of classical theory on parabolic equations and Black & Scholes theory on derivatives.

Expected learning outcomes (according to Dublin Descriptors)	Knowledge and understanding: Acquiring of fundamental concepts for the analysis of markets and contingent claims. Applying knowledge and understanding: Applying the theories acquired during the course for the study of stock markets. Making judgements: Ability to evaluate the reliability of the results obtained from the models. Communication: Acquiring the proper language of the financial markets. Lifelong learning skills: Acquire an appropriate study method, supported by text consultation and resolution of exercises and questions periodically proposed during the course.

Course program

1-Preliminaries. Convolution. Fourier transform. Absolute continuity of Lebesgue's integral. Passing to the limit below the integral sign. Lusin Theorem. Integration on unlimited domains.
2-Cauchy problem for parabolic equations with constant coefficients. Space C ^ {1,2}. Parabolic operators with constant coefficients. Classic solution of a parabolic equation. Classic solutions to a Cauchy problem that satisfy the initial condition almost everywhere, in the classical sense and in the L ^ 1_ {loc} sense. Relationship between solutions of a homogeneous parabolic equation and solutions of the homogeneous equation with only the higher order derivatives. Fundamental solution of a parabolic operator with constant coefficients. Pole of the fundamental solution. Estimates on the Fundamental Solution. Space GC. Existence of classic solutions of a homogeneous parabolic equation. The existence of classic solutions of a homogeneous Cauchy problem with continuous initial assumption taken in the classical sense. The existence of classical solutions of a homogeneous Cauchy problem with locally summable initial data assumed in the sense of L ^ 1_ {loc}. Existence of classic solutions to a non-homogeneous Cauchy problem. Nonuniqueness of the classic solutions of a Cauchy problem with initial data attained almost everywhere. Sing of the product of semindefined matrices product mark. Uniqueness of the classic solutions of a

Cauchy problem with initial datum attained in classical sense. Uniqueness of the classic solutions of a Cauchy problem with initial data assumed in the sense of L ^ 1_ {loc}.

3-Cauchy problem for parabolic equations with variable coefficients. Hypothesis. Space C ^ \ alpha_P. Fundamental solution. Existence of classic solutions to a non-homogeneous Cauchy problem. Weak Comparison Principle. Comparison, uniqueness, boundedness, and stability of the classic solutions of a Cauchy problem with continuous initial data. Uniqueness of the classic solutions of a Cauchy problem that satisfies the initial data in L ^ 1_{loc}. The fundamental solution as density. Uniqueness of bounded from below classical solutions.

4-Elements of stochastic calculus. Stochastic processes adapted to satisfy the usual hypotheses. Real standard Brownian motions. First and second variation of a numeric function. Martingale spaces. Notable examples. Doob Mayer's theorem on the quadratic variation of a continuous martingale. Properties of martingale compared to their first variation. Itô Integral in L ^ 2, L ^ 2_ {loc}. Itô's integral in L ^ 2. Local martingale spaces. Sufficient conditions for a local martingale continue is a continuous martingale. Quadratic variation of a local martingale and a stochastic integral function. Ito processes. Drift and diffusion coefficient. Quadratic variation. Uniqueness of the representation of an Itô process. A necessary and sufficient condition for an Ito process to be a local martingale. Normality of an Ito process with drift and deterministic diffusion coefficient. Itô formula. Calculation of E (W_t ^ n), E (and ^ {\ sigma W_t}). Martingale and parabolic equations. A necessary and sufficient condition for a Itô process f (t, X_0 = \ mu t + \ sigma W_t) to be a local martingale. Brownian geometric motion. A necessary and sufficient condition for a geometric Brownian motion is a martingale.

5- Contingent claims and arbitrage. Contingent claims. Options. American, Asian, European Options. Payoff of an option. Straddle. Financial Options. Derivative Market Issues: Evaluation and Replication. Arbiters. Absence of arbitrage. Put - Call parity and European price pricing estimates. Neutral prices and probabilities of risk. Arbitrage Price. Market Completeness. A generalization of the put-call parity of European options.

6-Black & Scholes Model - European contingent claims. Market composition. Wallets or strategies. Value of a strategy. Self-financing strategies. Performance of a strategy. Characterization of self-financing strategies through discounted stock prices and portfolio value. Strategies or Markovian Portfolios. Uniqueness of the representation of their value. Characterization of Markovian self-financing portfolios through the Black & Scholes equation. Reducing Black & Scholes Equation to Heat Equation. European contingent claims on the stochastic underlying of a Black & Scholes market. Eligible strategies. Reproducible European contingent claims. Completeness and absence of arbitraries from the Black & Scholes model. Evaluating European options on the stochastic underlying of a Black & Scholes market. Continuous dividends. Seeing where Black & Scholes model parameters are continuous functions of time. Eligibility and absence of arbitrage. Non-arbitrage principle. Risk Market Price. Valuation of contingent claims whose underlying is not traded on the market. Coverage strategies. The Greeks. The greeks of plain vanilla options. Price estimates of plain vanilla options. Gaming vega hedging strategies.

7-Model Black & Scholes - Exotic contingent claims. Asian Options and Black & Scholes Equation. Reducing the Black & Scholes equation to a parabolic equation for options with arithmetic mean. Reducing the Black & Scholes equation for Asian contingent claims with geometric averages at the Kolmogorov equation. Price calculation of an UpOut-Out Call option. Price of a Lookback derivative.

Teaching methods:

Lectures and exercise sessions.

Auxiliary teaching:

Assessment methods: Written exam.

Testi di riferimento principali:

A. Friedman. Partial Differential Equations of Parabolic Type. Dover Books on Mathematics, New York, 2008.

G. Gilardi. Analisi due. Second Edition. McGraw-Hill Book Co., Milano, 1996.

O. A. Ladyzenskaja, V. A. Solonnikov, N. N. Ural'ceva. Linear and Quasi-linear Equations of Parabolic Type. Translations of Mathematical Monographs, vol. 23, American Mathematical Society, Providence, 1968.

A. Pascucci. Calcolo stocastico per la finanza. Springer-Verlag, Milano, 2008.

E. Rosazza Gianin e C. Sgarra. Esercizi di Finanza Matematica, 1^a edizione. Springer-Verlag, 2007.

S. Shereve. Stochastic Calculus for Finance II. Springer-Verlag, 2004.

W. Rudin. Real and complex analysis. McGraw-Hill Book Co., New York, 1970.