Academic subject: Elements of Advanced Analysis 1					
Degree Class: L–35 – Scienze Matematiche		**Degree Course:** Mathematics		**Academic Year:** 2020/2021	
		Kind of class: mandatory		**Year:** 3	**Period:** 1
				ECTS: 7 divided into **ECTS lessons:** 6 **ECTS exe/lab:** 1	
Time management, hours, in–class study hours, out–of–class study hours lesson: 48 exe/lab.: 24 in–class study: 72 out–of–class study: 103					
Language: Italian	**Compulsory Attendance:** no				
Subject Teacher: Lorenzo D'Ambrosio	**Tel:** +39 080 5442692 **e–mail:** lorenzo.dambrosio@uniba.it	**Office:** Department of Mathematics Room 16, III Floor	**Office days and hours:** Tuesday 11–13. Other days and times by appointment.		
Prerequisites: Mathematical knowledge which usually is acquired during the first two years of a degree of L–35 class. Especially: classical analysis of one and several variables, general topology, linear algebra.					
Educational objectives: Acquiring language and techniques of modern analysis, especially measure theory, L^p spaces, Hilbert spaces, basic complex analysis in one variable.					
Expected learning outcomes (according to Dublin Descriptors)	**Knowledge and understanding:** Acquiring fundamental concepts in modern analysis and of elementary complex analysis. Acquiring basic mathematical proof techniques. **Applying knowledge and understanding:** The acquired theoretical knowledge is useful in large part of mathematics and its applications. **Making judgements:** Ability to analyze the consistency of the logical arguments used in a proof. Problem solving skills should be supported by the capacity in evaluating the consistency of the found solution with the theoretical knowledge. **Communication:** Students should acquire the mathematical language and formalism that are necessary to read and comprehend textbooks, to expound the acquired knowledge, and to describe, analyze and solve problems. **Lifelong learning skills:** Acquiring suitable learning methods, supported by text consultation and by solving the exercises and questions periodically suggested during the course.				

Course program

Real Analysis

1. Measure and abstract integration theory: σ–algebras, measurable sets and functions – elementary properties of the measure – integration of positive functions and complex–valued functions – sequences of integrals: Monotone Convergence Theorem, Fatou Lemma, Dominated Convergence Theorem – series of integrals – completion of a measure – Severini–Egoroff theorem – Vitali's convergence theorem.

2. Lebesgue measure in $\mathbf{R}^N$: simple sets, Lebesgue outer and inner measure – Lebesgue measurable sets – existence of non–Lebesgue measurable sets in R^N – positive translation–invariant Borel measures – Lebesgue measure and linear transformations: the geometric meaning of the determinant.

3. L^p spaces: Jensen, Hölder and Minkowsky inequalities – completeness of L^p spaces – continuity properties of measurable functions in R^N: Lusin's theorem – density of $C_c(R^N)$ into $L^p(R^N)$ – density of $C_c(R^N)$ into $C_0(R^N)$.

4. Elementary theory of Hilbert spaces: definition, Schwarz inequality, triangle inequality – existence of the element of smallest norm for closed convex sets – orthogonal projections – Riesz representation theorem in Hilbert spaces – the best approximation theorem – orthonormal sets, characterization of maximal orthonormal sets, existence of maximal orthonormal set – Bessel and Parseval identities, the isomorphism between H e $l^2(A)$ – the space $L^2(T)$ and the Fourier series – the spaces $H^s(T)$ e $H^s(T^N)$ and the embedding theorems into C(T) e $C(T^N)$ - Applications to differential equations and to the plane isoperimetric inequality.

Complex Analysis

5. Introduction to holomorphic function theory: complex differentiability: properties, geometric meaning – olomorphy and differentiability – Cauchy–Riemann equations and corollaries – some elementary holomorphic functions: complex exponential, complex trigonometric functions, multivalued functions and selections, complex logarithm, complex power – curves, paths, contours – a summary about differential forms – omothopy – simply connected sets – closed and exact differential forms – path integral – primitives of complex functions – holomorphic functions and differential forms – carachterization of the existence of primitives of complex functions – complex power series: convergence radius, uniform convergence, Cauchy–Hadamard theorem – Abel–Dirichlet test – Abel's theorem – Cauchy product – analytic functions – analiticity of the Cauchy integral.

6. Cauchy Theorem and analiticity of holomorphic functions: winding number theorem – Goursat theorem – existence of local primitives – Cauchy formula – analyticity of holomorphic functions – Morera's theorem – Cauchy formula for derivatives – Cauchy estimates for derivatives – fundamental theorem of algebra – Liouville's theorem for bounded holomorphic functions and generalizations – Morera–Weierstrass theorem – calculus of integrals.

7. Zeros of holomorphic functions and properties of harmonic functions: theorem about the zeros of holomorphic functions and corollaries – uniqueness of analytic continuation – real analytic functions – relationship between holomorphic and harmonic functions – mean value property – Pizzetti's formula – characterization of sub–harmonic and super–harmonic functions by means of their mean value – Liouville's theorem for positive harmonic functions and generalizations – maximum principle for sub–harmonic functions – Mean value theorem for holomorphic functions – maximum modulus principle, minimum modulus principle.

8. Residue Theorem and applications: isolated singularities – Laurent series – theorem about Laurent series developability – classification of isolated singularities and characterizations – Picard's theorem (only statement) – residues – calculus of the residue at a pole – residues theorem – Cauchy's theorem (general case) – Jordan's lemma – applications to integral calculus, series, differences equations – meromorphic functions – logarithmic index theorem – Rouché theorem and corollaries – open mapping theorem for holomorphic functions – inverse function theorem for holomorphic functions.

Teaching methods:
Lectures and exercise sessions.

Auxiliary teaching:
Didactic material available at
Istituzioni di Analisi Superiore
https://lorenzodambrosio.altervista.org/blog/didattica/istituzioni-analisi-superiore/

Assessment methods:
Oral exam.

Bibliography:

For the whole course: W. RUDIN, *Real and Complex Analysis,* McGraw–Hill Book Company

For the construction of Lebesgue measure in R^N: N. FUSCO, P. MARCELLINI & C. SBORDONE, *Analisi Matematica due,* Liguori

For analysis in one complex variable:

G. GILARDI, *Analisi 3,* Ed. Mc Graw–Hill; S. LANG, *Complex Analysis,* Springer–Verlag

Other didactic material (see above).