General information		
Academic subject	Riemannian Geometry	
Degree course	LM-40 - Mathematics	
Academic Year		
European Credit Transfer and Accumulation System (ECTS)		7
Language	Italian	
Academic calendar (starting and ending date)	II semester (28th February 2022 – 27th May 2022)	
Attendance		

Professor/ Lecturer	
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Tutoring (time and day)	Tutoring takes place by appointment to be agreed by email.

Syllabus	
Learning Objectives	Knowledge of the most important results in Riemannian geometry, with special attention to Hermitian and contact geometry.
Course prerequisites	Basic knowledge in differential geometry: differentiable manifolds, tangent and cotangent spaces, tangent bundle. Tensorial algebra and tensorial calculus. Elements of Riemannian geometry.
Contents	**Complex vector spaces**. Complexification of real vector spaces and real linear maps. Complexification of the dual vector space. Complex structures on real vector spaces. Canonical complex structure on R^{2n}. C-linear maps between complex vector spaces. GL(n,C) as subgroup GL(2n,R). Positively oriented bases in a complex vector space. Complexification of a complex vector space. Vectors and forms of type (1,0) and (0,1). Decomposition of the complexified exterior algebra. **Almost complex manifolds.** Almost complex structures on differentiable manifolds. Adapted frames. Orientability of almost complex manifolds. Canonical almost complex structure on C^n. Vector fields and 1-forms of type (1,0) and (0,1). Decomposition of the complexified exterior bundle. The Nijenhuis tensor associated to an almost complex structure: necessary and sufficient conditions for its vanishing. **Complex manifolds.** Holomorphic functions and Cauchy-Riemann equations. Complex manifolds. Holomorphic maps between complex manifolds. Examples: complex projective space, S^2, biholomorphism between S^2 e CP^1. Canonical almost complex structure on a complex manifold. Adapted local frames and coframes. The Newlander-Nirenberg Theorem. Differential operators on complex manifolds. Complex structures on Riemannian oriented surfaces. Characterizations of holomorphic functions. Holomorphic vector fields and holomorphic 1-forms. Real

	holomorphic vector fields. **Almost Hermitian manifolds.** Hermitian inner products on complex vector spaces. The standard Hermitian inner product on C^n. Hermitian matrices. Orthonormal bases and unitary matrices. Real part and imaginary part of a Hermitian inner product. 2-fundamental form and orthonormal J-bases. U(n) as a subgroup of SO(2n). Complexification of a Hermitian inner product and its fundamental 2-form. Almost Hermitian manifolds. Existence of Hermitian metrics. C^n as a Hermitian manifold. Adapted local orthonormal frames. Non-degeneracy of the fundamental 2-form. The Levi-Civita connection: covariant derivatives of the almost complex structure and the fundamental 2-form. The Nijenhuis tensor of an almost Hermitian manifold. Some classes of almost Hermitian manifolds. Holomorphic sectional curvature. **Kähler manifolds.** Definition and characterization of Kähler manifolds. Kähler structures on Riemannian oriented surfaces. Riemannian curvature properties for a Kähler manifold. Kähler manifolds with constant sectional curvature. Kähler manifolds with constant holomorphic sectional curvature. Hermitian metrics in complex coordinates. Characterization of Kähler metrics. Kähler potential. The Bergman metric on the complex disk. The Fubini-Study metric on the complex projective space. Classification of Kähler manifolds of constant holomorphic sectional curvature. **Symplectic manifolds.** Existence of canonical bases for a skew-symmetric form. Symplectic forms on a vector space and symplectic bases. Characterization of symplectic forms. Almost symplectic and symplectic manifolds. Almost Hermitian manifolds as almost symplectic manifolds. The standard symplectic structure on R^{2n}. Darboux Theorem for symplectic manifolds. Existence of an almost complex structure on a symplectic manifold. **Contact and almost contact manifolds.** Contact elements on a manifold. Contact structure on a manifold as distribution of contact elements. 1-forms that (locally and globally) define a contact structure. Dimension and orientability of a differential manifold equipped with a contact structure. Standard contact structure on R^{2n+1}. (ϕ, ξ, η)-structures, compatible metrics, adapted orthonormal frames. Almost contact metric manifolds. Normal almost contact structures: definition and characteriziation. Almost contact structure on an orientable hypersurface of an almost Hermitian manifold. Example: the structure induced on the sphere S^{2n+1} by the canonical Hermitian structure of R^{2n+2}. Normal almost contact manifolds: definition and properties. *D*-homothetic deformations. Rank of an almost contact structure. **Some classes of almost contact metric manifolds.** Contact metric manifolds, K-contact manifolds, Sasaki manifolds, cosymplectic manifolds, nearly cosymplectic manifolds, quasi Sasaki manifolds, β-Kenmotsu manifolds: definitions,

	characterizations, and relations with classes of almost Hermition manifolds. **Riemannian curvature properties**. Curvature identities for contact metric manifolds. ξ-sectional curvature of K-contact manifolds. Properties of the Ricci tensor for K-contact manifolds. Characterization of Sasaki manifolds by means of the Riemannian curvature. Contact metric manifolds with constant sectional curvature. Nullity conditions for the Riemannian curvature of a contact metric manifold, (κ, μ)-contact manifolds. ϕ-sectional curvature, and manifolds with (pointwise) constant ϕ-sectional curvature. Sasakian space forms. η-Einstein and Sasaki-Einstein manifolds.
Books and bibliography	– Blair D.E.: Riemannian Geometry of Contact and Symplectic Manifolds, Springer (2010) – Cannas Da Silva, A.: Lectures on Symplectic Geometry, Springer (2008) – Kobayashi, S., Nomizu K.: Foundations of Differential Geometry vol.1, Wiley-Interscience (1996) – Moroianu, Lectures on Kähler geometry. London Mathematical Society Student Texts, 69. Cambridge University Press, Cambridge, 2007.
Additional materials	

Work schedule			
Total	Lectures	Hands on (Laboratory, working groups, seminars, field trips)	Out-of-class study hours/ Self-study hours
Hours			
150	*52*	*8*	*90*
ECTS			
7	*6,5*	*0,5*	
Teaching strategy	Lectures and exercises.		
Expected learning outcomes			
Knowledge and understanding on:	Acquiring results in recently investigated research fields of Riemannian geometry, allowing to comprehend advanced textbooks and recent publications.		
Applying knowledge and understanding on:	Acquiring proof techniques in Hermitian and contact geometry, together with the knowledge of fundamental examples.		
Soft skills	• *Making informed judgments and choices* Ability to analyze the consistency of mathematical arguments, under the formal, logical and technical point of view. Students should become able to prove autonomously properties dealing with the program topics. • *Communicating knowledge and understanding* Students should acquire the mathematical language and formalism necessary to the comprehension and the exposition of concepts and results concerning the studied theory. • *Capacities to continue learning*		

	Improve learning methods acquired during previous years, through the practice in exposing results, solving problems and bibliographic search.

Assessment and feedback	
Methods of assessment	Oral exam.
Evaluation criteria	*At the end of the course the following points will be evaluated:* • *Knowledge and understanding:* knowledge of the fundamental notions of Hermitian and contact geometry, together with the ability to prove their properties. • *Applying knowledge and understanding:* Ability to solve problems and illustrate the acquired notions in specific examples. • *Autonomy of judgment*: Ability to evaluate the consistency of the arguments used in a proof, and to compare alternative proofs. Capacity to ask questions and propose solutions. • *Communication skills*: Ability to expose theorems, proofs, questions, through a suitable language and mathematical formalism. • *Capacities to continue learning:* Ability to consult advanced texts and scientific articles, also written in English.
Criteria for assessment and attribution of the final mark	The final grade is out of thirty. The exam is passed when the grade is greater or equal to 18/30.
Additional information	