General information	
Academic subject	***GEOMETRY 3***
Degree course	*Year 2 Period 1*
Academic Year	*2021-22*
European Credit Transfer and Accumulation System (ECTS)	8
Language	*Italian*
Academic calendar (starting and ending date)	*27/9/2021-23/12/2021*
Attendance	*Strongly recommended*

Professor/ Lecturer	
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Tutoring (time and day)	By appointment

Syllabus	
Learning Objectives	Acquiring the basic concepts in Projective Geometry and in the theory of conics and quadrics.
Course prerequisites	Mathematical knowledge which is usually acquired during the first year of the degree in Mathematics. Especially: linear algebra, affine and Euclidean spaces.
Contents	**Projective spaces.** The projective space P(V) associated to a vector space V. Projective spaces KP^n. Projectively independent points. Subspaces; lines, planes, hyperplanes. Intersection of subspaces and projective subspace joining a finite family of subspaces. Projective Grassmann formula. Cartesian and parametric equations of a subspace. Linear system of hyperplanes containing a given subspace. Projective transformations of KP^n, projective transformation group. Points in general position and relative characterization. Existence and uniqueness theorem for projective transformations. Models of projective spaces over a field K. Coordinated systems. Field expanded with the element at infinity and its canonical structure as a model of projective line. Structure of projective space induced on a set by transport by means of a bijection. Projective completion S(A) of an affine space A with the addition of directions. Coordinated systems admissible on S(A) deduced from affine frames. Projective transformations between projective spaces: characterization, properties, equations, existence and uniqueness theorem. Projective transformations transform subspaces into subspaces. Canonical structure of projective space induced on a subspace. Dual of a projective space. Linear systems of hyperplanes considered as projective subspaces of the dual space. Examples: pencils of lines and pencils of planes. Projection from a point to a pencil of lines in a projective plane. Characterization of the projective subspaces of S(A) which are not contained in the hyperplane at infinity as extensions S(E) of affine subspaces E of A.

	Determination of the equations of the projective completion of an affine subspace starting from its equations in an affine frame. Canonical extension of an affinity to a projective transformation. Canonical isomorphism between the group of affinities and the group of projective transformations that preserve the hyperplane at infinity. **Projective geometry in one dimension.** Non-homogeneous projective coordinate (projective abscissa); relevant special cases. The bilinear equation of a projective transformation between projective lines. Cross ratio. Harmonic conjugate point of an ordered triple of points. Characterization of the projective transformations between projective lines as bijective maps preserving the cross ratio. Projective transformations that transform an ordered set of four distinct points into another. Elliptic, parabolic, hyperbolic projectivities. Involutions. Fixed points of a projectivity. The characteristic of a hyperbolic projectivity. Existence and uniqueness of the hyperbolic projectivity of assigned fixed points joined and assigned characteristic. The circular involution on a proper pencil of lines in the projective completion of a Euclidean plane. **Projective hyperquadrics.** Quadratic forms and symmetric bilinear forms: signature, radical. Hyperquadrics of a geometric space of dimension $n \geq 1$. Support of a hyperquadric. The set of hyperquadrics is a projective space of dimension $n(n + 3)/2$. Rank of a hyperquadric. Image of a hyperquadric through a projective transformation. Projective classification of hyperquadrics in the complex case. Maximum dimension of a projective subspace contained in a complex non-degenerate hyperquadric (statement only). Index of a hyperquadric of a real projective space and its geometric meaning. Classification theorem in the real case. The case of conics and quadrics. Definition of elliptic quadric and hyperbolic quadric. Intersection between a hyperquadric and a projective subspace; relative positions between a line and a hyperquadric. Singular points of a hyperquadric, radical. Characterization of the hyperquadrics which coincide with their radical. Structure of degenerate hyperquadrics that do not coincide with their radical: projective cones; examples in dimension two and three. Conjugated points. Polar hyperplane of a non-singular point; polarity. Discussion of the intersection between a hyperquadric and the polar hyperplane of a point. Tangent hyperplanes. Tangents conducted from a point to a conic. Elliptic points and hyperbolic points of a non-degenerate real quadric. The involution of the conjugate points on a secant or external line to a hyperquadric. Lines which are conjugated with respect to a conic, involution of the conjugated lines passing from a fixed a point. **Affine properties of hyperquadrics.** The center of a non-degenerate hyperquadric of the projective completion of an affine space. Central hyperquadrics. Definition of ellipse, hyperbola, parabola and their characterizations. Diameters of a conic, conjugated diameters. Asymptotes of a hyperbola. Conjugation of diameters. Definition of ellipsoid, hyperboloid and paraboloid. Notion of affine hyperquadric and

	its projective closure. The bijective correspondence between affine hyperquadrics and projective hyperquadrics that do not contain the hyperplane at infinity. Affine equivalence criterion for hyperquadrics (statement only). Canonical equations of affine hyperquadrics. The case of conics and quadrics. Cones and cylinders. Practical methods to classify a real conic or a real quadric from the projective or affine point of view. The notion of pencil of hyperquadrics. Description of some relevant types of conic pencils. **Metric properties of hyperquadrics.** Hyperspheres viewed as special hyperquadrics of a Euclidean space. Principal hyperplanes of a non-degenerate hyperquadric. Axes of a conic and its vertices. Standard equation of a conic in a suitable orthonormal frame. Round quadrics. The foci of a conic. Eccentricity.
Books and bibliography	M. Beltrametti, E. Carletti, D. Gallarati, G. Monti Bragadin: *Lezioni di Geometria analitica e proiettiva*, Bollati Boringhieri, 2003. M. Berger: *Geometry II*, Universitext, Springer-Verlag, 1987. E. Casas-Alvero: *Analytic Projective Geometry,* EMS Textbooks in Mathematics. European Mathematical Society (EMS), 2014. E. Sernesi: *Geometria 1,* Bollati Boringhieri, 1994. E. Fortuna, R. Frigerio, R. Pardini: *Geometria proiettiva, problemi risolti e richiami di teoria*, Springer-Verlag, Collana Unitext, 2011.
Additional materials	Auxiliary didactic material available on the teacher's homepage (https://www.dm.uniba.it/members/lotta) and on the Microsoft Teams platform of the course.

Work schedule			
Total	Lectures	Hands on (Laboratory, working groups, seminars, field trips)	Out-of-class study hours/ Self-study hours
Hours			
70	*40*	*30*	*130*
ECTS			
8	*5*	*3*	
Teaching strategy	Lectures and exercise sessions.		
Expected learning outcomes			
Knowledge and understanding on:	Acquiring fundamental concepts about classical geometrical topics using a modern language.		

Applying knowledge and understanding on:	The acquired knowledge has a wide spectrum of applications, both in the field of pure mathematics and other scientific disciplines, for example in computer science (3D graphics, design, robotics, computer vision, etc.).

Soft skills	• *Making informed judgments and choices*
	  ○ Ability in developing new methods which are useful in problem solving.
	• *Communicating knowledge and understanding*
	  ○ *A*cquiring the mathematical language and formalism which are necessary to analyze and solve problems.
	• *Capacities to continue learning*
	  ○ Acquiring suitable learning methods and ability of relating the main concepts occurring in various mathematical disciplines.

Assessment and feedback	
Methods of assessment	*Written and oral exams*

Evaluation criteria	• *Knowledge and understanding*
	  ○ Knowledge of the fundamental concepts and notion of Projective Geometry, of the basic results of the theory and of their proofs.
	• *Applying knowledge and understanding*
	  ○ Ability to solve problems concerning classical geometric objects, like lines, planes, pencils, conics and quadrics, espacially by using the interplay between projective and affine geometry.
	• *Autonomy of judgment*
	  ○ Critical thinking about the topics discussed during the lessons, capacity of developing proofs of some facts autonomously.
	• *Communicating knowledge and understanding*
	  ○ Clear exposition of concepts, constructions, of theorems and their proofs.
	• *Communication skills Mastery* of abstract mathematical reasoning.
	• *Capacities to continue learning*
	  ○ Mastery of linear algebra within a geometrical framework.

Criteria for assessment and attribution of the final mark	*The written exam must be passed before giving the oral exam. Final mark should be at least 18/30. Students are allowed to give the oral examination within a different session after passing the written one, according to the general rules established by the Consiglio Interclasse di Matematica.*
Additional information	