General information		
Academic subject	**Geometry 2**	
Degree course	L-35 - Mathematics	
Academic Year	1st	
European Credit Transfer and Accumulation System (ECTS)		8
Language	Italian	
Academic calendar (starting and ending date)	2nd semester (28th February 2022 – 27th May 2022)	
Attendance	According to didactic regulations	

Professor/ Lecturer	
Name and Surname	Giulia Dileo
E-mail	giulia.dileo@uniba.it
Telephone	+39 080 5442679
Department and address	Department of Mathematics, 2nd floor, room 35
Virtual headquarters	https://www.dm.uniba.it/members/dileo - Teams
Tutoring (time and day)	Tutoring takes place by appointment to be agreed by email. A weekly appointment will be announced at the beginning of the semester.

Syllabus	
Learning Objectives	Acquiring fundamental notions in affine and Euclidean geometry.
Course prerequisites	Basic knowledge of linear algebra: matrix calculus, linear systems, vector spaces, linear maps and bilinear forms.
Contents	**Euclidean vector spaces.** Scalar products on real vector spaces. The norm of a vector. Orthogonal and orthonormal vectors. Orthogonal complement of a vector subspace. Orthonormal bases. Gram-Schmidt process. Angle between two vectors. Selfadjoint endomorphisms. Spectral Theorem. Unitary operators. Orthogonal matrices. Rotations and reflections. **Affine spaces.** Geometric vectors. Affine spaces associated to vector spaces: definition, elementary properties and first examples. Vector spaces as affine spaces. Affine frames and coordinate systems. Change of affine frames. Orientation of a real affine space. Barycenter of weighted points. **Affine subspaces.** Affine subspaces and their direction. Affine space structure induced on an affine subspace. Characterization of affine subspaces via barycenters. Affine subspace spanned by a finite number of points: definition and characterizations. Affinely independent points. Collinear and coplanar points. Ratio of three collinear points. Intersection of affine subspaces. Parallel subspaces. The affine subspace spanned by two subspaces. Affine Grassmann identity. Coplanar lines. Parametric and cartesian equations of an affine subspace. *Affine geometry in dimension 2*. Parametric equations and cartesian equation of a line. Coordinate axes. Parallel lines and intersection of lines. Sheaves of lines.

Affine geometry in dimension 3. Parametric and cartesian equations of a plane. Parallel planes. Intersection of planes. Parametric and cartesian equations of a line. Coordinate axes and planes. Parallel lines. Parallelism between a line and a plane. Coplanar lines. Sheaves of planes.

Euclidean spaces.
Euclidean space associated to a Euclidean vector space. Cartesian frames and cartesian coordinates. Change of cartesian frames. Distance between two points. Euclidean affine subspaces. Angles between two lines. Convex angle between two oriented lines. Orthogonal lines. Orthogonal subspaces. Orthogonal projection of a point onto a subspace. Distance of a point to a subspace. Euclidean line.

Euclidean geometry in dimension 2. Angles between lines. Orthogonal lines. Angles between a line and coordinate axes. Angular coefficient of a line. Distance of a point to a line. Circles.

Euclidean geometry in dimension 3. Angles between lines. Orthogonal lines. Angles between a line and coordinate axes. Angles between planes and orthogonal planes. Angle between a line and a plane. Orthogonality between a line and a plane. Distance of a point to a plane. Distance of a point to a line. Minimum distance between lines. Spheres and circles. Surfaces of revolution: cones and cylinders.

Affine maps and affine transformations.
Affine maps: definition and first properties. Characterization of affine maps via barycenters. Example: projection parallel to a direction onto an affine subspace. Images of affine subspaces under affine maps. Affine transformations. The affine group. Existence and uniqueness of affine transformations. Equations of an affine transformation. Affinely equivalent sets. Translations: definition, characterization and equations. Fixed points of an affine transformation. Affine transformations with a fixed point. Decomposition of an affine transformation. Homotheties and symmetries.

Isometries.
Isometries of a Euclidean space: definition and characterization. Isometries and angles between lines. Existence and uniqueness of isometries. Isometric sets. Equations of an isometry. Direct isometries and opposite isometries. Rotations and reflections. Decomposition of isometries. Reflections with respect to a hyperplane. Decomposition of isometries into reflections with respect to hyperplanes. Isometries of the Euclidean plane: translations, rotations, reflections, glide reflections. Chasles' Theorem. Rotations in the 3-dimensional Euclidean space. Hints on the classification of isometries in the 3-dimensional Euclidean space.

Affine and Euclidean conics.
Conics in the real or complex affine plane. Equation of a conic. Affine and Euclidean classification of conics, and their canonical forms.

Books and bibliography	– E. Abbena, A.M. Fino, G.M. Gianella, *Algebra lineare e geometria analitica*, Aracne. – S. Abeasis, *Algebra lineare e Geometria*, Zanichelli. – M. Audin, *Geometry*, Universitext, Springer. – M. Berger, *Geometry I*, Universitext, Springer. – G. Campanella, *Affinità, isometrie, proiettività*, Aracne. – E. Sernesi, *Geometria 1*, Bollati Boringhieri. – M.I. Stoka, *Corso di Geometria*, Ed. Cedam Padova.
Additional materials	

Work schedule			
Total	Lectures	Hands on (Exercises)	Out-of-class study hours/ Self-study hours
Hours			
200	*40*	*30*	*130*
ECTS			
8	*5*	*3*	
Teaching strategy	Lectures and exercises. Exercise sheets will be provided.		
Expected learning outcomes			
Knowledge and understanding on:	Acquiring fundamental concepts in affine and Euclidean geometry. Acquiring basic mathematical proof techniques.		
Applying knowledge and understanding on:	Students should become able to use the acquired theoretical knowledge in solving problems.		
Soft skills	• *Making informed judgments and choices* Ability to analyze the consistency of the logical arguments used in a proof. Problem solving skills should be supported by the capacity in evaluating the consistency of the found solution with the theoretical knowledge. • *Communicating knowledge and understanding* Students should acquire the mathematical language and formalism necessary to read and comprehend textbooks, to explain the acquired knowledge, and to describe, analyze and solve problems. • *Capacities to continue learning* Acquiring suitable learning methods necessary to read and understand texbooks dealing with the program topics.		

Assessment and feedback	
Methods of assessment	Joint exam with the course Geometry 1, consisting in a written test and an oral examination. The written exam consists in solving exercises, the oral exam consists in the exposition of definitions, statements and proofs, and the discussion of specific examples.

Evaluation criteria	*At the end of the course the following points will be evaluated:* • *Knowledge and understanding:* knowledge of the fundamental concepts in affine and Euclidean geometry, together with the capacity to state and prove related properties; capacity to show the acquired notions in specific examples. • *Applying knowledge and understanding:* knowledge of how to use the acquired theoretical notions in solving exercises of affine and Euclidean geometry, including: scalar products, orthonormal bases, orthogonal complement, selfadjoint operators; parametric and cartesian equations of affine and Euclidean subspaces, and their geometric properties (parallelism, intersection, coplanar lines, orthogonality, angles, distances); equations of affine transformations and isometries, their properties and classification problems; equations of circles, spheres, surfaces of revolution; affine and Euclidean classification of conics. • *Autonomy of judgment*: capacity in evaluating the consistency of the logical arguments used in a proof. Problem solving skills, coherently with the acquired theoretical knowledge. • *Communication skills*: capacity in the exposition of definitions, statements and proofs, and in presenting solutions of exercises in suitable mathematical language and formalism. • *Capacities to continue learning*: capacity in consulting textbooks, in finding logical links and solving exercises.
Criteria for assessment and attribution of the final mark	The final grade is out of thirty. The exam is passed when the grade is greater or equal to 18/30.
Additional information	