

Academic subject: Geometry 1.			
Degree Class: L-35, Scienze Matematiche		Degree Course: Mathematics	
		Academic Year: 2020/2021	
		Kind of class: mandatory	
		Year: 1	Period: 1
		ECTS: divided into ECTS lessons: 5 ECTS exe/lab/tutor: 3	
Time management, hours, in-class study hours, out-of-class study hours lesson: 40 exe/lab/tutor: 55 in-class study: 95 out-of-class study: 105			
Language: Italian		Compulsory Attendance: no	
Subject Teacher: Maria Falcitelli		Tel: 0805442844 e-mail: maria.falcitelli@uniba.it	
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		Office days and hours: by appointment.	
Prerequisites: Basic notions in Mathematics taught in high school.			
Educational objectives: Acquiring basic notions of linear algebra which will be used in the following courses.			
Expected learning outcomes (according to Dublin Descriptors)		<p>Knowledge and understanding: Acquiring fundamental concept, such as matrices, linear systems, vector spaces, linear maps and bilinear forms.</p> <p>Applying knowledge and understanding: The acquires theoretical knowledge is useful in many branches of Mathematics, such as affine and projective Geometry.</p> <p>Making judgements: Ability to analyze the consistency of a proof. Ability in problem solving.</p> <p>Communication: Acquiring mathematical basic language and formalism.</p> <p>Lifelong learning skills: Acquiring suitable learning methods, solving problems related to the contents of the course.</p>	

Course program

Algebraic structures

Binary operations and algebraic structures. Groups, subgroups and elementary properties. Rings, integral domains, fields, subfields. Homomorphisms of groups and fields. The kernel and image of a homomorphism. Complex numbers and the field of complex numbers. The ring of polynomials over a field. Algebraically closed fields.

Matrices and linear systems.

Matrices with elements over field. Transpose of matrix. Diagonal, symmetric and skew-symmetric matrices. Sum and product of matrices. The group $GL(n, k)$ and its subgroups. Rank of a matrix and properties. Determinant of a square matrix and its properties. Theorem of Binet. Laplace's rule. Cramer's rule. Theorem of Rouchè- Capelli. Systems of linear equations. Homogeneous systems.

Vector spaces.

Vector spaces over a field K : properties and fundamental examples. Polynomials in one indeterminate. Operations on polynomials and the vector space of polynomials. The vector space of matrices. Vector subspaces, examples. Intersection, sum, direct sum of vector subspaces. Supplementary subspaces. Vector space generated by n vectors. Finitely generated vector spaces. Linearly independent and dependent vectors. Bases of a vector space. Components of a vector with respect to a basis. Dimension of a vector space. Grassmann identity. Existence of a supplementary subspace of a vector subspace. Changes of bases. Orientations.

Linear maps

Linear maps: characterization and properties. Fundamental examples. The kernel and image of a linear map. Existence and uniqueness of linear maps. Characterization of monomorphisms and isomorphisms. Linear forms and dual space. Matrices associated to a linear map. Linear map associated to a matrix.

Endomorphisms.

Definition of endomorphism. Eigenvectors, eigenvalues and eigenspaces of an endomorphism. The characteristic polynomial. Algebraic and geometric multiplicity of an eigenvalue. Diagonalizable endomorphisms and matrices. Diagonalization criteria.

Bilinear forms

Definition of bilinear form. Symmetric and skew-symmetric bilinear forms. Matrices associated to a bilinear form. Congruent matrices. Orthogonal vectors. Orthogonal complement of a vector subspace. Fourier coefficient. Orthogonal bases. Diagonalization of a symmetric bilinear form. Symmetric bilinear forms on a complex vector space. Quadratic forms. Sylvester's Theorem. Signature of real quadratic form: semidefinite, definite and indefinite forms.

Teaching methods:

Lectures and exercise sessions.

Auxiliary teaching:

Tutorial activities.

Assessment methods:

Written and oral exam, jointly with Geometry 2.

Bibliography:

E. Sernesi, Geometria 1, Boringhieri

A. Facchini, Algebra e Matematica discreta, Zanichelli

E. Abbena, A.M. Fino, G.M. Gianella, Algebra lineare e Geometria analitica, Vol. I, II, Aracne.

E. Schlesinger, Algebra lineare e Geometria, Zanichelli

L. Muri, E. Schlesinger, Esercizi di algebra lineare e geometria, zanichelli.