Stampare su carta intestata del CdS

General information		
Academic subject	***Numerical Calculus 2***	
Degree course	*Mathematics (**Degree Class:** L-35)*	
Academic Year	*2021/2022*	
European Credit Transfer and Accumulation System (ECTS)	7 (5 lectures + 2 exe/lab)	
Language	*Italian*	
Academic calendar (starting and ending date)	*Second semester*	
Attendance	*no*	

Professor/ Lecturer		
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Department and address	*Dip. Matematica piano IV, stanza 2*	*Dip. Matematica piano II, stanza 12*
Virtual headquarters	*Microsoft Teams*	*Microsoft Teams*
Tutoring (time and day)	Information available at the url: https://www.dm.uniba.it/members/iavernaro/ricevimento	Information available at the url: https://www.dm.uniba.it/members/vacca/ricevimento

Syllabus	
Learning Objectives	*Acquiring knowledge about numerical methods and programming techniques in the context of interpolation, data fitting, numerical integration, numerical solution of ordinary differential equations.*
Course prerequisites	*The knowledge gained in the course "Numerical Claculus 1", programming in Matlab, classical analysis of one and several variables, fundamental linear algebra.*
Contents	***1. INTERPOLATION.*** *The polynomial interpolation problem: existence and uniqueness theorem. Undetermined coefficient method: computational cost and stability issues. Lagrange formula and barycentric formulations. Remainder term in the polynomial interpolation. Convergence properties. Stability properties: Lebesgue function and Lebesgue constant. Newton divided differences interpolation formula: Newton basis and divided differences. Newton-Gregory forward difference formula. Runge's phenomenon. Chebyshev nodes and their optimality property. Chebychev-Lobatto nodes and related barycentric formula. Convergence results for differentiable and analytic functions. Hermite interpolation: extensions of Lagrange and Newton interpolation formulae, remainder theorem and computational properties. Linear splines: construction of a basis, stability, remainder and convergence properties. Cubic spline interpolation: extra boundary conditions, construction and implementation; hints about convergence and minimal curvature property. Generalized polynomials and Haar condition. Trigonometric interpolation: DFT and IDFT algorithms, computational complexity and implementation; some applications to the signal processing.* ***2. APPROXIMATION.*** *Overdetermined linear systems. Least squares approximation: problem definition, existence and uniqueness theorem. Least Squares fitting-polynomial. Linear regression line. Coefficient of determination. QR factorization of a rectangular matrix by means of*

	Householder elementary transformations. Use of QR factorization for solving least squares problems. Singular Value Decomposition: existence result and fundamental properties. Solution of the least squares problem through the SVD decomposition. Pseudoinverse and its properties. Conditioning of the least squares problem. Bidiagonalization of a matrix by means of Householder elementary transformations. Algorithm for the SVD computation by means of Givens rotations. Some applications of the SVD: computation of the rank and the 2-norm of a matrix; best low-rank approximation of a matrix; regularizing an ill-conditioned linear system; digital image compression; latent semantic analysis; principal component analysis. Least squares problem in L2([a,b]): problem definition, use of an orthogonal basis, Bessel's inequality and Parseval's identity. Error estimation and implementation aspects. Orthogonal polynomials: Gram-Schmidt ortogonalization process, three-term recurrence relation, property of the roots. Rodrigues' formula and study of the family of Legendre polynomials, first and second kind Chebyshev polynomials, Laguerre and Hermite polynomials. Truncated Fourier series. Padé rational approximation. 3. ***NUMERICAL INTEGRATION.*** *Interpolation quadrature formulae, degree of precision and error analysis. Rectangle, trapezoidal and Simpson rules: definition and error analysis. Newton-Cotés formulae. Conditioning of the problem and stability of an interpolation quadrature formula. Composite trapezoidal and Simpson's rules, error analysis. Recursive algorithm with automatic step control based on the trapezoidal Simpson's rules. Higher-order formulae, weights positivity and convergence properties. Gauss-Legendre, Radau and Lobatto formulae. Hints on adaptive integration techniques.* 4. ***NUMERICAL METHODS FOR INITIAL-VALUE PROBLEMS.*** *One-step methods: definition, consistency and convergence analysis. Explicit and implicit Euler methods, implicit midpoint and trapezoidal methods. Collocation methods: definition and link to Runge-Kutta methods. Existence and uniqueness of the collocation polynomial. Order analysis. Gauss, Radau, Lobatto collocation methods.* 5. ***PROGRAMMING ENVIRONMENT FOR SCIENTIFIC COMPUTING.*** *The implementation of the algorithms introduced in the course will be carried out in Matlab, which is a scientific computing environment equipped with a number of built-in functions and programming instructions. A particular attention will be paid to the study of the behavior of the solutions of a given problem in finite arithmetic.*
Books and bibliography	• *L.N. Trefethen, Approximation Theory and Approximation Practice SIAM, 2013.* • *G.H. Golub, C.F. Van Loan, Matrix computations. Fourth edition. Johns Hopkins Studies in the Mathematical Sciences. Johns Hopkins University Press, Baltimore, MD, 2013.* • *R. Bevilacqua, D. Bini, M. Capovani, O. Menchi, Metodi Numerici, Zanichelli, Bologna, 1992.* • *E. Hairer, S.P. Nørsett, G. Wanner, Solving Ordinary Differential Equations I. Nonstiff Problems. Springer Series in Comput. Mathematics, Vol. 8, Springer- Verlag 1987, Second revised edition 1993.* • *D. Bini, M. Capovani, O. Menchi, Metodi numerici per l'algebra lineare, Zanichelli.*

	• *K.E. Atkinson, An introduction to numerical analysis, Wiley, 1989* • *D.M. Young, R.T. Gregory, A survey of numerical mathematics, Vol. I, Dover, 1988*
Additional materials	*Handouts, notes and Matlab codes will be made available through an e-learning platform. Log-in information will be provided at the course starting days*

Work schedule			
Hours			
Total 70	Lectures 40	Hands on (Laboratory, working groups, seminars, field trips) 30	Out-of-class study/self-study 105
ECTS			
7	*Lectures: 5*	*Exe/lab: 2*	
Teaching strategy			
	Lectures and exercise sessions. Exercise sessions in the Computer Centre		
Expected learning outcomes			
Knowledge and understanding on:	○ Acquiring a knowledge about the most important numerical methods able to solve mathematical real-world problems, with particular reference to data-fitting, numerical integration and initial value problems. ○ Understanding and being able to explain issues related to the use of a computer for solving the above mentioned mathematical problems.		
Applying knowledge and understanding on:	○ The solution of mathematical problems by means of suitable algorithms enjoying good stability properties and low cost implementation. ○ encoding and testing numerical algorithms and consistently interpreting computer results.		
Soft skills	• *Making informed judgments and choices* ○ Being able to detect a proper numerical method for solving a given mathematical problem among those analyzed during the lectures. • *Communicating knowledge and understanding* ○ Being able to provide rigorous definitions of the analyzed mathematical problems and to discuss the related numerical methods, outlining their most prominent features. • *Capacities to continue learning* ○ Capability of studying and solving problems similar, but not necessarily equivalent, to those faced during the teaching activities.		

Assessment and feedback	
Methods of assessment	*The exam consists of an oral test which includes a discussion of the Matlab codes prepared by the student. Before taking the exam, the student must deliver to the teacher a paper (in pdf or power-point format) in which he presents the numerical experiences and the results obtained.*
Evaluation criteria	• *Knowledge and understanding* ○ Identification of the fundamental properties of the methods, with particular reference to the hypothesis of applicability and computational efficiency. ○ Ability to compare methods that solve the same problem. • *Applying knowledge and understanding*

	◦ Discussion of the codes and examples performed; correct interpretation of the results obtained. • *Autonomy of judgment* ◦ Collection and processing of data, and interpretation of the results obtained. • *Communication skills* ◦ Clarity, also in terms of formalism, in the description and coding of the numerical methods, as well as ability to effectively present the numerical tests carried out. • *Capacities to continue learning* ◦ Numerical implementation and discussion of more elaborate application problems than those presented during the lectures.
Criteria for assessment and attribution of the final mark	The final score takes into account both the theoretical discussion of the methods studied and their correct implementation on the computer. The quality of the examples produced and the ability to illustrate them are also duly considered.
Additional information	