General information		
Academic subject	***Advanced course in Mathematical Analysis 1***	
Degree course	*Mathematics (LM-40)*	
Academic Year	*Second*	
European Credit Transfer and Accumulation System (ECTS)		7 divided into: ECTS lessons :6,5 ECTS exe 0,5
Language	*Italian*	
Academic calendar (starting and ending date)		*September 27th – December 23th, 2021*
Attendance	*No compulsory attendance*	

Professor/ Lecturer	
Name and Surname	Giusi Vaira
E-mail	giusi.vaira@uniba.it
Telephone	+39 080 544 2706
Department and address	*Department of Mathematics, Via E. Orabona 4, Room 16, Floor IV*
Virtual headquarters	*Teams, Analisi Superiore 1, access code* **fntkzlr**
Tutoring (time and day)	Days and times have to be arranged by email

Syllabus	
Learning Objectives	*To acquire knowledge and techniques of modern mathematical analysis, especially compact and locally compact topological spaces and continuous function spaces on them, compactness criteria in continuous function spaces, density theorem in continuous function spaces, Radon measure on locally compact spaces, Hausdorff measure and self-similar sets and basic notion of calculus of variations, especially for what concern finite perimeter sets and isoperimetric problems.*
Course prerequisites	*Mathematical knowledge which is usually acquired with a degree of L-35 class. Especially: classical analysis of one and several variables, metric spaces and Banach spaces, elements of general topology, abstract measure theory and integration.*
Contents	***COMPACT AND LOCALLY COMPACT TOPOLOGICAL SPACES*** *Topological spaces. Compact and locally compact topological spaces. Alexandrov compactification. Urysohn theorem. The theorem on the finite partition of unity. Locally compact spaces countable at infinity. Locally compact spaces with countable bases. Separable topological spaces.* ***CONTINUOUS FUNCTION SPACES*** *The Banach space C(X) of all (real-valued) continuous functions on a compact space X. The linear space of all continuous functions with compact support on a locally compact Hausdorff space. Continuous functions converging at infinity on a locally compact space. The Banach spaces C_0(X) and C_*(X) of all continuous functions vanishing at infinity, resp. converging at infinity, on a locally compact space X.* ***COMPACTNESS THEOREMS*** *Tychnoff theorem. Equicontinuous subsets of mapping. Equicontinuous subsets of linear mappings. Examples and properties. Uniformly convergent sequences of mapping and equicontinuity. The Ascoli-Arzelà theorem. Application to the study of*

	integral operators. Compact maps. The Banach theorem on the weak compactness of the closed balls of a separable Banach space.* ***DENSITY THEOREMS*** *Stone-Weierstrass type theorems for lattice subspaces and for subalgebras of C(X, R) and C(X, C), X compact space. The Weierstrass approximation theorem (algebraic and trigonometric form). Stone-Weierstrass theorems in C_0(X, R) and C_0(X, C), X locally compact. Applications.* ***FIXED POINT THEOREMS*** *Introduction to fixed point theorems. Banach fixed point theorem. Brouwer fixed point theorem. Compact mappings between normed spaces. The Schauder fixed point theorem. Some applications to integral equations and to ordinary differential equations. The Leray-Schauder principle and a priori estimates. Applications to partial differential equations.* ***POSITIVE LINEAR OPERATORS AND POSITIVE LINEAR FORMS ON CONTINUOUS FUNCTION SPACES; RADON MEASURES*** *Positive linear forms and positive linear operators on continuous function spaces. Positivity and continuity. Radon measures on locally compact spaces. Radon measures with finite support. Regularity and approximation theorems. The dual space of C_0(X, R).* ***HAUSDORFF MEASURE*** *Hausdorff measure and its properties. Hausdorff dimension of a set. Cantor set. Hausdorff dimension of continuous functions belonging to Holder spaces. Rectifiable curves and Hausdorff dimension. Self-similar sets and the calculus of Hausdorff dimension. Sierpinski trangle, von Koch curve. Steiner symmetrization and properties. Isodiametric inequality. Relation between Lebesgue and Hausdorff measure. Density property with respect to Hausdorff measure. Measure of the graph of a Lipschitz function. Area and coarea formulas for Lipschitz functions.* ***INTRODUCTION TO CALCULUS OF VARIATIONS*** *Direct methods of calculus of variations. Classical functional: Eulero-Lagrange equations, Du Bois-Reymon equation. Convessity methods. Fermat principle for geometric optics. Brachistocrone problem. Functional of the gradient. Functionals on Sobolev spaces: convessity and lower semicontinuity in W^{1, p}. Existence of minimum in W^{1, p}. Examples. Functions of bounded variations and properties. Finite perimeter sets. Properties of perimeter function. Isoperimetric inequality. Isoperimetric problems as variational problems when the minimum of the functional is achieved with symmetry properties. Rearrangments, symmetrizations and application to variational problems. Symmetrizations and partial differential equations.*
SierBooks and bibliography	*[1] H. Bauer, Measure and Integration Theory, De Gruyter Series Studies in Mathematics, 26, De Gruyter & Co. Berlin, New York, 2001* *[2] G. Choquet, Lecture on Analysis, vol. I, W. A. Benjamin Inc., New York, 1969*

	[3] L.C. Evans, Partial Differential Equations, AMS, Providence, 1998 *[4] G. B. Folland, Real analysis, J. Wiley & Sons Inc., New York, 1999* *[5] M. Giaquinta, S. Hildebrandt, Calculus of variations I, Springer, 2006* *[6] W. Rudin, Real and complex analysis, McGraw-Hill Inc., New York, 1987*
Additional materials	*Notes on calculus of variations.*

Work schedule			
Total	Lectures	Hands on (Laboratory, working groups, seminars, field trips)	Out-of-class study hours/ Self-study hours
Hours			
175	*52*	*8*	*115*
ECTS			
7	*6,5*	*0,5*	

Teaching strategy	
	Lectures and exercise sessions.
Expected learning outcomes	
Knowledge and understanding on:	○ To acquire fundamental concepts and tools in modern mathematical analysis. ○ To acquire basic mathematical proof techniques. ○ Ability to apply the results to study differential equations.
Applying knowledge and understanding on:	The acquired theoretical knowledge is useful in great part of mathematics and its applications: especially to the study of partial differential equations that describe, for example, classical problems in Geometry and in Mathematica Physics.
Soft skills	• *Making informed judgments and choices* ○ Ability to analyze the consistency of the logical argument used in a proof. ○ Problem solving skills should be supported by the capacity in evaluating the correct methods required for studying mathematical problems (mainly of variational type) modern and complex. • *Communicating knowledge and understanding* ○ Students should acquire the mathematical language and formalism necessary to read and comprehend textbooks, to explain the acquired knowledge, and to describe, analyze and solve complex problems. • *Capacities to continue learning* ○ To acquire suitable learning methods, supported by text consultation and, sometimes, by scientific articles, and by solving the exercise and questions periodically suggested throughout the course.

Assessment and feedback	We will evaluate the knowledge of the theoretical notions, the ability to apply the acquired knowledge to questions and problems. Furthermore, the mathematical language used for the presentation will be also object of evaluation.
Methods of assessment	*Oral exam.*
Evaluation criteria	• *Knowledge and understanding*

	○ **Minimum level:** knowledge of the principal results of the course and some proof of them. ○ **Intermediate level**: knowledge of the results of the course and of the principal proofs. ○ **Upper level:** knowledge of all results of the course and of the proofs. • *Applying knowledge and understanding* ○ **Minimum level:** ability to apply the results to basic problems proposed during the exam. ○ **Intermediate level:** ability to apply the results to problems proposed during the exam. ○ **Upper level:** ability to apply the results to complex problems proposed during the exam. • *Autonomy of judgment* ○ **Minimum level**: to know how to apply the results to basic problems proposed during the course through correct and logic arguments. To know how to prove some theorems following a logic-deductive reasoning. ○ **Intermediate and upper level**: to know how to apply the results to advanced and complex problems proposed during the exam through correct and logic arguments. To know how to prove the principal theorems following a logic-deductive reasoning. • *Communicating knowledge and understanding* **For all the levels:** to know the results and understand the relation between them. • *Communication skills* ○ **For all the levels:** to show the knowledge of the correct mathematical terminology and to present the arguments with a formal and correct language. • *Capacities to continue learning* ○ **For all the levels:** ability to make relations between the various arguments and the way to apply them.
Criteria for assessment and attribution of the final mark	*The final mark is between 18/30 and 30/30 (or 30 cum laude). The exam is passed when the mark is greater than 18. If the student shows the knowledge of all the results and the ability to apply them to various and complex mathematical problems it is possible to have 30 cum laude.*
Additional information	