General information		
Academic subject	Mathematical Analysis no. 3	
Degree course	Mathematics	
Academic Year	Second	
European Credit Transfer and Accumulation System (ECTS)		8
Language	Italian	
Academic calendar (starting and ending date)	First semester (September 27, 2021 – December 23, 2021)	
Attendance	Discretionary	

Professor/ Lecturer	
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Virtual headquarters	Microsoft Teams Code: 63uef9r
Tutoring (time and day)	In-person or online. Days and times have to be arranged by e-mail

Syllabus	
Learning Objectives	This course is aimed at providing further language and techniques of classical Mathematical Analysis, in particular sequences and series of functions, basics of metric and normed spaces, basics of functions in several variables.
Course prerequisites	In order to understand and be able to apply most of the techniques taught in this course, students have to master the basic knowledge given at the first year of a degree course in Mathematics. In particular, classical Mathematical Analysis for real single variable functions (differential and integral calculus), basics of Linear and Matrix Algebra, Analytic Geometry.
Contents	**Sequences and series of functions** Sequences of functions: pointwise and uniform convergence. Theorem on the interchange of limits. Theorem on the continuity of the limit function. Cauchy criterion for uniform convergence. Theorem on the interchange of limit and integral. Theorem on the interchange of limit and derivative. Series of functions: pointwise, uniform and absolute convergence. Cauchy criterion for uniform convergence of series. Theorem on the term-by-term integration. Theorem on term-by-term differentiation. Properties of the sum function. Power series and set of convergence. Radius of convergence and its characterization. Cauchy-Hadamard Theorem. D'Alembert Theorem. Abel Theorem. Power series obtained by integration or derivation and their convergence radius. Taylor series and analytic functions. Sufficient condition for the expansion of a function in Taylor series. The binomial series. Noteworthy Taylor series. Fourier series. Dirichlet's criterion on the pointwise convergence of a Fourier series. Theorem on the uniform convergence of a Fourier series. **Metric spaces, normed spaces, the Euclidean space IR^N** Metric spaces and distances. Equivalent metrics. Topology of a metric space: open balls, neighborhoods, open and closed subsets. Interior, limit, boundary and isolated points of a set. Connected sets. Bounded sets and diameter. Convergent sequences and Cauchy sequences. Complete metric spaces. Compact sets. Limits and continuity in metric spaces. Lipschitz-continuous mappings and Banach fixed-point theorem in metric spaces.

	Vector spaces. Linearly independent vectors and bases. Dual space. The linear space IR^N and its standard basis. Linear functionals on IR^N. Linear maps from IR^N to IR^m and matrices. Normed spaces and Banach spaces. Equivalent norms. Noteworthy normed spaces. Norm induced metric and norm associated to an inner product. Bounded functions. The linear space IR^N: noteworthy norms and inner product. Young, Hölder and Minkowski inequalities. Compact subsets of IR^N and Heine-Borel Theorem. The (generalized) Weierstrass theorem. Uniformly continuous mappings. The (generalized) Cantor theorem. Line segments, polygons, convex sets, star-shaped sets, connected sets in IR^N and their characterization. Limits of functions in IR^N. Continuous functions: properties and main theorems. **Differential calculus for functions in several variables** Real functions in several variables: partial derivatives and their properties. Gradient vector. Higher order partial derivatives. Schwarz theorem and Hessian matrix. Directional derivative or Gateaux derivative. Differentiability and the Fréchet derivative. Differentiable functions: properties. Sufficient condition for the differentiability of a function. Vector-valued functions in a single variable. Chain rule for differentiable functions. Lagrange Mean Value Theorem. Functions with zero gradient. Taylor formulas. Homogeneous functions and Euler's homogeneous functions theorem. Functions defined by integrals: continuity and differentiability. Vector-valued functions from IR^N to IR^m and the Jacobian matrix. Differentiability and related theorems. The higher-dimensional chain rule. Square matrices and their eigenvalues. Quadratic forms and symmetric matrices. Definite, semidefinite matrices and their characterizations. Local extrema for real functions in several variables. Fermat Theorem. Necessary condition of the second order. Second partial derivative test for local extrema. Constrained and absolute extrema. **Implicit functions and Dini's Theorem** Introduction to the implicit function problems. Two-dimensional Dini's implicit function theorem. Implicit differentiation. The higher-dimensional Dini's implicit function theorem. Dini's implicit function theorem for systems. Constrained extrema problems. Lagrange multipliers method.
Books and bibliography	• N. Fusco - P. Marcellini – C. Sbordone, *Lezioni di Analisi Matematica Due*, Zanichelli Ed., Bologna, 2020. (Capitoli 1, 2, 3, 11) • P. Marcellini – C. Sbordone, *Esercitazioni di Analisi Matematica Due, Prima parte*, Zanichelli Ed., Bologna, 2018. • P. Marcellini – C. Sbordone, *Esercitazioni di Analisi Matematica Due, Seconda parte*, Zanichelli Ed., Bologna, 2018.
Additional materials	It is recommended to complete textbooks with notes taken at lesson. The recommended textbook can be replaced by any other books of Mathematical Analysis which cover the topics of the program. If using notes found on internet, a careful check about their author is strongly recommended.

Work schedule			
Total	Lectures	Hands on (exercises)	Out-of-class study hours/ Self-study hours
Hours			
200	40	30	130
ECTS			
8	5	3	

Teaching strategy	
	Classroom lectures which include exercises whose purpose is to make the student acquire the ability to apply theoretical concepts. Due to the ongoing health emergency, teaching will take place according to the Academic Senate's resolutions.
Expected learning outcomes	
Knowledge and understanding on:	Fundamental concepts in the classical Mathematical Analysis and related mathematical proof techniques.
Applying knowledge and understanding on:	The acquired mathematical knowledge is useful in large part of Mathematics and its applications.
Soft skills	• *Making informed judgments and choices* Students have to develop critical thinking so to distinguish between essential and nonessential assumptions, moreover they have to identify the most appropriate mathematical tools for solving a given problem and have to realize the limitations of techniques and methods. • *Communicating knowledge and understanding* Students have to acquire both language and advanced mathematical formalism so to be able to retrieve useful information from Maths textbooks, to discuss mathematical results and to describe, to analyze and to solve given problems. • *Capacities to continue learning* Students have to acquire the ability to study and understand mathematical topics, and also to retrieve useful information from Maths textbooks so to apply them for solving problems.

Assessment and feedback	
Methods of assessment	The final grade comes from: • a written exam (about two hours long), • an oral exam. The written test is preparatory to the oral exam. Its result will be communicated either by e-mail or through platform ESSE3.
Evaluation criteria	Students must: • solve exercises on sequences and series of functions, • study real functions in several variables: limits and local, constrained, global minimum and maximum points, • study implicit fuctions. Moreover, they must also be able to prove theoretical results, to distinguish between essential and nonessential assumptions, to discuss mathematical notions in a rigorous way, to contextualize mathematical topics.
Criteria for assessment and attribution of the final mark	The final grade, out of thirty, takes into account both written and oral test. The exam is passed if the final grade is greater than or equal to 18/30. For taking the oral exam, a student must pass the written test with a grade which is greater than or equal to 15/30.
Additional information	
	The course is carried out in collaboration with Prof. Elvira Mirenghi e-mail: elvira.mirenghi@uniba.it telephone: +39 0805442675 department and address: Department of Mathematics (Floor II, Room 29) Tutoring is in-person or online. Days and times have to be arranged by e-mail.
	Attendance is strongly recommended.