Stampare su carta intestata del CdS

General information	
Academic subject	**Functional Analysis**
Degree course	Mathematics
Academic Year	2021-2022
European Credit Transfer and Accumulation System (ECTS)	7
Language	Italian
Academic calendar (starting and ending date)	
Attendance	No. Optional but recommended attendance

Professor/ Lecturer	
Name and Surname	Genni Fragnelli
E-mail	genni.fragnelli@uniba.it
Telephone	+39 080 544 2687
Department and address	Department of Mathematics via E. Orabona 4
Virtual headquarters	
Tutoring (time and day)	By appointment via email

Syllabus	
Learning Objectives	Acquiring language and basic tools concerning functional spaces, representation theorems, operator theory and operator semigroups, with applications to some classes of partial differential equations
Course prerequisites	Mathematical knowledge which usually is acquired during the first two years of a degree of L-35 class. Especially: classical analysis of one and several variables, normed spaces, general topology, linear algebra.
Contents	**1. Recalls on topological spaces, metric spaces, Banach spaces and Hilbert spaces** Recall on the main properties of topological spaces, metric spaces, Banach spaces and Hilbert spaces. Baire spaces. The Baire Lemma, the Riesz Lemma and the Riesz Theorem. **2. Sequence spaces and functional spaces** The space l^p, $1\leq p\leq +\infty$, c_0, c_{00}. Hölder continuous functions, Lipschitz continuous functions, $W^{1,p}(a,b)$, $W_0^{1,p}(a,b)$ and $W^{k,p}(a,b)$ spaces, with $1\leq p\leq +\infty$ and k in IN. Bounded variation functions and absolutely continuous functions. Strictly convex and uniformly convex spaces. The first Clarkson inequality. The Morawetz inequality. The Clarkson Theorem. Uniform convexity of Hilbert spaces. **3. Functional Theory** Functionals and properties. Linear and continuous functionals and associated hyperplanes. The Zorn Lemma. The analytic form of the Hahn-Banach Theorem and its consequences. Dual of a normed space and its properties. The Minkowski functional and its properties. The first and the second geometric form of the Hahn-Banach Theorem. Bidual spaces and reflexive spaces. The James Theorem. The Milman-Pettis Theorem. Reflexivity of Hilbert spaces and examples. The Theorem of Riesz-Frechet and its application to the dual spaces of $L^p(\Omega)$ and l^p, $1\leq p<+\infty$. The dual space of c_0. The dual spaces of $L^\infty(\Omega)$ and l^∞. **4. Weak topology and weak* topology** Weak topology in Banach space. Weak convergence and its properties. Weak convergence in Hilbert spaces. Weak convergence in finite dimensional spaces. Convex and closed sets in the weak topology and in the strong convergence.

	Weak* topology and weak* convergence. The Banach–Alaoglu–Bourbaki Theorem and the Kakutani Theorem. Reflexive spaces and weak* topology. The existence of minimum points for convex functions. Reflexive and separable spaces. The ball in the weak topology and in the weak* topology and its consequences. The Eberlein-Smulian Theorem. Completely continuous operators. Characterization of the weak convergence and the weak* convergence in $L^p(\Omega)$ and l^p spaces. The Severini Egorov Theorem and its consequences. The Radon-Riesz property for L^p and l^p spaces and its consequences. The Schur Theorem. **5. Linear and continuous operators** Linear and bounded operators and their properties. Linear operators defined in finite dimensional spaces. Norm of an operator. The Theorem of Neumann Series. The Banach-Steinhaus Theorem and its consequences. The Open Mapping Theorem and its applications. The Closed Graph Theorem. Unbounded linear operators. Closed operators. Adjoint operators and their properties. Closed range operators and their properties. Operators of finite rank. Approximable operators and their properties. Resolvent set, spectrum and point spectrum of an operator and their properties. **6. Compact operators** Compact operators and their properties. Approximation problem. The Schauder Theorem. The spectrum of a compact operator. The Fredholm operators. The Fredholm alternative. Compact embedding of Sobolev spaces. Completely continuous operators and compact operators. **7. Operators in Hilbert spaces** Orthonormal basis in separable Hilbert spaces. Hilbert – Schmidt operators. Bounded self-adjoint operators, monotone operators, idempotent operators and normal operators. Invertibility of a self-adjoint operator. Symmetric, self-adjoint and maximal monotone unbouded operators. Properties of the spectrum for a self-adjoint operator. Spectrum of a monotone and self-adjoint operator. Hilbert basis and eigenvectors of compact and self-adjoint operators. **8. Semigroups, Operators and Resolvents** Resolvent of an operator and Yosida approximation. Strongly continuous semigroup on Banach spaces, its generator and some examples. Properties of a generator. Cauchy problem and semigroups. The Hille-Yosida Theorem in Banach spaces and in Hilbert spaces. The self-adjoint case. Regularity of operator semigroups. The heat equation
Books and bibliography	H. BREZIS, Analyse fonctionnelle, Theorie et applications, 2e tirage, Masson 1987. H. BREZIS, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer, 2011. P. CANNARSA – T. D'APRILE, Introduzione alla teoria della misura e all'analisi funzionale. Springer, 2008. E. DIBENEDETTO, Real Analysis, Birkäuser, 2002. K.J. ENGEL - R. NAGEL, One-parameter Semigroups for Linear Evolution Equations, Graduate Texts in Mathematics 194, Springer, 2000. G. GILARDI, Analisi 3, McGraw-Hill, Milano 1994. G. GILARDI, Analisi Funzionale, McGraw-Hill, Milano 2014 J.A. GOLDSTEIN, Semigroups of Operators and Applications, Second Edition, Dover Publications, Inc. New York 2017. P.D. LAX, Functional Analysis, Wiley Interscience, 2002 M. MURATORI, F. PUNZO, N. SOAVE, Esercizi svolti di Analisi Reale e Funzionale, I Edizione, Società Editrice Esculapio, Bologna 2021
Additional materials	Books should be completed with the notes of the lessons. It is suggested not to take notes from Internet.

Work schedule	

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Total	Lectures	Hands on (Laboratory, working groups, seminars, field trips)	Out-of-class study hours/ Self-study hours
Hours			
175	52	8	*115*
ECTS			
7			

Teaching strategy	Lectures and exercise sessions
Expected learning outcomes	
Knowledge and understanding on:	Acquiring fundamental concepts and results in the setting of functional spaces and operator theory. Acquiring main tools and proof techniques.
Applying knowledge and understanding on:	The acquired theoretical knowledge finds many applications in several aspects of mathematics, including partial differential equations and related models.
Soft skills	• *Making informed judgments and choices* Ability to analyze the consistency of the logical arguments used in a proof, the problem solving skills and the ability to choose suitable mathematical tools consistent with the theoretical knowledge. • *Communicating knowledge and understanding* Acquiring the mathematical language and the formalism necessary to read and understand textbooks, to explain the acquired knowledge, and to describe, analyze and solve problems. • *Capacities to continue learning* Acquiring suitable learning methods, supported by consultation of texts and by solving exercises and problems related to the contents of the course.

Assessment and feedback	
Methods of assessment	The oral test is mandatory and is designed to assess the level of knowledge attained by the student on the theoretical contents of the course. The exam is passed if the final grade is at least 18/30.
Evaluation criteria	• *Knowledge and understanding* It will be evaluated the acquisition of the fundamental concepts and results in the setting of functional spaces and operator theory and the acquisition of the main tools and of the proof techniques • *Applying knowledge and understanding* It will be evaluated the acquired theoretical knowledge in several applications • *Autonomy of judgment* It will be evaluated the ability to analyze the consistency of the logical arguments used in a proof, the problem solving skills and the ability to choose the suitable mathematical tools consistent with the theoretical knowledge • *Communication skills* It will be evaluated the acquisition of the mathematical language together with the formalism and the preciseness necessary to explain the acquired knowledge, and to describe, analyze and solve problems. • *Capacities to continue learning* It will be evaluated the acquisition of suitable learning methods, supported by consultation of texts and by solving exercises and problems related to the contents of the course.
Criteria for assessment and attribution of the final mark	The students must be able to connect the arguments recognizing their consequences.

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Additional information	