Academic subject: Nonlinear Analysis

Degree Class: L-35 Mathematical Sciences	**Degree Course:** Mathematics	**Academic Year:** 2021/2022	
	Kind of class: Optional	**Year:** III	**Period:** 2
		ECTS: 7 divided into **ECTS lessons:** 6,5 **ECTS exe/lab/tutor:** 0,5	

Time management, hours, in–class study hours, out–of–class study hours

lesson: 52 exe/lab/tutor: 8 in–class study: 60 out–of–class study: 115

Language:	**Compulsory Attendance:**		
Italian	no		
Subject Teacher: Prof. Silvia Cingolani	**Tel:** +39-0805442660 **e–mail:** silvia.cingolani@uniba.it **Home page:** https://www.dm.uniba.it/members/cingolani	**Office:** Department of Mathematics Floor II, Room 11	**Office days and hours:** Days and times have to be arranged by email

Prerequisites:

In addition to the mathematical knowledge which usually is acquired during the first two years of a degree of L–35 class, they are required language and techniques of modern analysis such as basic theory of Hilbert spaces and L^p spaces.

Educational objectives:

Acquiring instruments of Functional Analysis and techniques of Nonlinear Analysis, which allow one to study some nonlinear differential equations, which solutions correspond to critical points of a suitable functional defined on a Banach space. Proving some abstract existence theorems in Critical Point Theory and furnishing applications to the study of linear and nonlinear variational problems coming from Physics, Geometry and Applied Sciences.

Expected learning outcomes (according to Dublin Descriptors)

Knowledge and understanding:
Acquiring fundamental concepts of variational methods, their related proof techniques and how to apply them for studying some differential equations.

Applying knowledge and understanding:
The acquired variational methods apply for studying some nonlinear differential equations which describe some classical problems in Geometry and in Mathematical Physics.

Making judgements:
Ability to analyze the consistency of the logical arguments used in a proof. Problem solving skills should be supported by the capacity in evaluating the correct methods required for studying nonlinear differential equations with variational structure.

Communication:
Students should acquire the mathematical language and formalism necessary to read and comprehend textbooks, to explain the acquired knowledge, and to describe, analyze and solve variational problems.

Lifelong learning skills:
Acquiring suitable learning methods, supported by text consultation and by solving some nonlinear model differential equations.

Course program

Function spaces. Background knowledge on spaces of C^k functions and on Lebesgue spaces. Elements of distribution theory. Weak convergence. Sobolev spaces and their main properties. Sobolev Embeddings. Critical exponent. Poincarè Inequality.

Linear problems. Elements of spectral theory. Symmetric and self-adjoint operators. Friedrichs extension. Self-adjoint extension and its spectral properties for the Laplace operator with omogeneous Dirichlet boundary conditions. Weak solutions of elliptic boundary value problems. Regularity theorems.

Differential calculus in Banach spaces. Fréchet and Gâteaux derivatives. Theorem of Total Differential. Properties and examples of differentiable functionals. Higher order derivatives. Critical points and local extrema. Fermat Theorem in Banach spaces. Weierstrass Theorem in Banach spaces.

Nonlinear differential problems. Nemytskii operators on Sobolev spaces. Weak solutions and variational principles for some nonlinear differential problems. Hamilton's principle of least action. Weierstrass Theorem and existence of weak solutions. Applications to semilinear elliptic equations. Ekeland’s variational principle. Critical points of functionals on manifolds and some problems with constraints: non-homogeneous elliptic problems, dynamical systems on manifolds and their trajectories joining two fixed points, nonlinear eigenvalue problems. Variational problems with unbounded functionals. The Palais-Smale condition and its variants. Deformation Theorems. Mountain Pass Theorem. Three Solutions Theorem. Applications to some nonlinear differential equations on bounded and unbounded domains. Pohozaev Identity. Lack of compactness. Group Action. Principle of symmetric criticality. Subcritical Sobolev inequalities. Brezis-Lieb Lemma. Nonlinear Schrodinger Equations in Quantum Mechanics.

Teaching methods:
Lectures and exercise sessions

Auxiliary teaching:
Some notes on the lectures.

Assessment methods:
Oral exam

Bibliography:

- R.A. Adams & J.J.F. Fournier, “Sobolev Spaces” (2nd Ed.), Academic Press, Amsterdam, 2003
- Ambrosetti & G. Prodi, “A Primer of Nonlinear Analysis”, Cambridge University Press, Cambridge, 1993
- M. Badiale, E. Serra, “Semilinear Elliptic Equations for Beginners”, Springer-Verlag 2010
- H. Brezis, “Functional Analysis, Sobolev Spaces and Partial Differential Equations”, Springer, New York, 2011
- D. Costa, “An Invitation to Variational Methods in Differential Equations”, Birkhäuser, Basel, 2007
- J. Mawhin, M. Willem, “Critical Point Theory and Hamiltonian Systems”, Springer-Verlag, Berlin, 1989
- M. Struwe, “Variational Methods. Applications to Nonlinear Partial Differential Equations and Hamiltonian Systems” (4th Ed.), Ergeb. Math. Grenzgeb. (4) 34, Springer–Verlag, Berlin, 2008