Academic subject: Numerical Methods and Modelling				
Masters Program: LM-40- Mathematics		**Program:** Mathematics	**Academic Year:** 2020/2021	
Required course			**Year:** 1	**Semester:** 1
			ECTS: 7 divided into **Lectures**: 4 **Exercise session:** (exe/lab/tutoring) 3	
Time management, hours, in–class study hours, out–of–class study hours Lectures: 40; Exe/lab/tutor: 32. Total in class: 72. Total home-study : 103 hours.				
Language: Italian	**Required Attendance:** no			
Instructor: Luciano Lopez	**Tel:** +39 080 5442655 **e–mail:** luciano.lopez@uniba.it	**Office:** Department of Mathematics Room 15 , Floor 2	**Office days and hours:** Wednesday 11-13	
Prerequisites: Basic knowledge about systems of differential equations, Linear Algebra and numerical analysis, bases of programming. All requirements are achieved in the Bachelor class L-35.				
Educational goals: Acquiring methods and techniques of applied mathematics to the simulation of continuous and discrete models of evolution. Ability of mathematical modeling of simple phenomena.				
Expected learning outcomes (according to Dublin Descriptors)	**Knowledge and understanding:** Acquiring fundamental concepts of modelling and dynamical systems; equilibrium points, stability, limit cycles, long-term behaviors. **Applying knowledge and understanding:** Acquiring methods for simulating discrete and continuous models and interpretation of results. **Making judgements:** Ability to evaluate the correspondence of the models with the reality that they want to represent and possibly the ability to modify them. **Communication:** Acquiring advanced mathematical concepts in the description of models and their simulation **Lifelong learning skills:** Acquiring appropriate learning methods through systematic use of books, solution of exercises, and simulation of models by computers.			

Course program

1. **DISCRETE DYNAMICAL SYSTEM.**

First order difference equations and their solutions. Theory of linear difference equations od order k. Homogeneous difference equations. Computation of solutions. Characteristic polynomial: single roots and multiple roots. Operator p(E) and its properties. Computation of particular solutions. Equilibria of difference equations and their stability. Formal series. Variation of constants formula. Linear systems of difference equations. Stability of solutions of difference equations. Matrix functions and their properties. Asymptotic study of A^n and exp(tA). Discrete models: Cobweb simple and complex; Lesley model of population dynamic; Natchez indian.

2. **CONTINUOUS DYNAMICAL SYSTEMS**

Autonomous linear systems of ODES: Principal matrix solution and its properties. Matrix exponential. Continuity of solutions with respect to the initial condition and concept of stability. Invariant subspaces: stable, unstable and central space. Examples: planar linear systems; node, saddle point, focus and center.

Autonomous nonlinear systems. Properties of the flow. Equilibria and concepts of asymptotic stability, stability and instability. Equilibria and linearizations. Lyapunov functions. Periodic orbits and limit cycles. Long time behavior of solutions. Continuous models: harmonic oscillator with damping and forcing. Duffing oscillator with damping and forcing. Nonlinear pendulum. Van der Pol. Lotka-Volterra, competing species, Darwin.

3. RUNGE KUTTA METHODS

Forward Euler: local and global error. Linear stability. FE applies to the harmonic oscillator. Runge Kutta method of order 2. Local and global error. Linear stability and behavior of solutions of the harmonic oscillator. Spurious fixed points. Runge Kutta methods general theory: consistency, convergence, local and global error, upper bounds. Verification of order of convergence in function of the discretization stepsize. Linear stability: stability function and region of absolute stability.

Teaching methods: On line lectures and exercise sessions.

Evaluation methods and grading: Oral exam and computer simulation of models

Bibliography:

V. Lakshmikantham, D. Trigiante, Theory of difference equations: numerical methods and applications, Academic Press Inc, 1988.

D.G: Luemberger, Introduction to dynamic systems, J. Weley and Sons, 1979.

M. Braun, Differential Equations and Their Applications**:** An Introduction to Applied Mathematics: An Introduction to Applied Mathematics. Springer, 1983.

L. Perko, Differential Equations and Dynamicla Systems, Springer, 1991.

J.D. Lambert, Numerical Methods for Ordinary Differential Systems: The Initial Value Problem, Wiley and Sons