Academic subject: Advanced Geometry 2				
Degree Class: LM-40 – Matematica		**Degree Course:** Mathematics	**Academic Year:** 2020/2021	
		Kind of class: Mandatory/Optional depending on the Curriculum	**Year:**	**Period:** 2
			ECTS: 7 divided into **ECTS lessons:** 6,5 **ECTS exe/lab/tutor:** 0,5	

Time management, hours, in–class study hours, out–of–class study hours
lesson: 52 exe/lab/tutor: 8 in–class study: 60 out–of–class study: 115

Language: Italian	**Compulsory Attendance:** no		
Subject Teacher: Antonio Lotta	**Tel:** +390805442656 **e–mail:** antonio.lotta@uniba.it	**Office:** Department of Mathematics Room 7, Floor II	**Office days and hours:** By appointment

Prerequisites: Basic knowledge of smooth manifolds and Lie groups. Elementary notions about Riemannian metrics: Levi-Civita connection, geodesics, curvature.

Educational objectives: Acquiring knowledge of some advanced topics of modern Riemannian Geometry, especially concerning homogeneous spaces and some important results concerning the relationship between curvature and topology, providing the necessary background for further study of the subject at Phd level.

Expected learning outcomes (according to Dublin Descriptors)

Knowledge and understanding: Acquiring some fundamental concepts and proof techniques in modern differential geometry.

Applying knowledge and understanding: The acquired theoretical knowledge is useful in great part of mathematics and of theoretical physics.

Making judgements: Ability to comprehend and rework the proofs of meaningful mathematical results. Ability to test some general facts on specific examples.

Communication: Students should acquire the mathematical language and formalism necessary to read and comprehend advanced textbooks and specialized literature on the subject and to explain the acquired knowledge.

Lifelong learning skills: Acquiring suitable learning methods, supported by text consultation and by elaborating on questions periodically suggested during the course.

Course program

Homogeneous spaces. Actions of Lie groups on manifolds. Fundamental vector fields. Free and proper actions. Quotient of a manifold by a regular equivalence relation: theorem of Godement. Quotient of a Lie group by a closed subgroup. Examples.

Riemannian homogeneous spaces. Homogeneous Riemannian manifolds. Examples. Isotropy representation. Criterion for the existence of invariant metrics on a homogeneous space. Existence of bi-invariant metrics on compact Lie groups. Homogeneous spaces with irreducible isotropy representation. The Levi-Civita connection of an invariant metric in the reductive case. Killing fields and their characterization. Nomizu’s formula for the curvature tensor of a reductive homogeneous Riemannian manifold. Naturally reductive homogeneous spaces. Normal metrics, Samelson's theorem. Invariant metrics on Lie groups. Geodetic completeness of Riemannian homogeneous spaces.

Isometries of compact Riemannian manifolds. Outline of the Myers-Steenrod theorem on the group of isometries of a Riemannian manifold. Maximum dimension of the isometry group. Divergence theorem. Killing fields on compact manifolds: Bochner's theorem. Application to the homogeneous case.

Jacobi fields. Exponential map. Normal neighbourhoods and geodesic balls. Uniformly normal neighbourhoods. Jacobi fields. The Gauss Lemma.

Locally symmetric spaces and space forms. Geodesic symmetries. Characterization of locally symmetric Riemannian manifolds. Cartan's theorem on the existence of local isometries between locally symmetric spaces. Global version of Cartan's theorem. Classification of space forms (Theorem of Hopf).
Riemannian symmetric spaces. Examples. Canonic representation of a symmetric space Riemannian as a homogeneous reductive space; Cartan decomposition. A simply connected, complete locally symmetric is globally symmetric. Curvature and Ricci tensor. Compact and non-compact type spaces and sign of the sectional curvatures. Decomposition theorem.

Curvature and topology. Concept of distance for a connected Riemannian manifold. Minimization properties of geodesics. The Hopf-Rinow and Cartan-Hadamard theorems. Kobayashi's theorem on the topology of homogeneous Riemannian manifolds with nonpositive curvature and negative definite Ricci tensor. Myers' theorem.

Teaching methods: Lectures and exercise

Auxiliary teaching:

Assessment methods: Oral exam

Bibliography:

1) S. Kobayashi, K. Nomizu: Foundations of differential geometry. Vol. II, John Wiley & Sons, Inc., New York, 1969.
2) J.M. Lee: Riemannian manifolds. Graduate Texts in Mathematics 176, Springer-Verlag, New York, 1997.
3) B. O'Neill: Semi-Riemannian geometry. Academic Press, San Diego, 1983.
4) M. Postnikov: Geometry VI. Riemannian geometry. Encyclopaedia of Mathematical Sciences 91, Springer-Verlag, Berlin, 2001.