<table>
<tr><td colspan="6">Academic subject: Advanced Geometry 1</td></tr>
<tr><td colspan="2">Degree Class: LM-40</td><td colspan="2">Degree Course:<br>Mathematics</td><td colspan="2">Academic Year:<br>20202021</td></tr>
<tr><td colspan="2"></td><td colspan="2">Kind of class:<br>Mandatory/optional depending on the curriculum</td><td>Year:<br>2</td><td>Period:<br>1</td></tr>
<tr><td></td><td></td><td colspan="2"></td><td colspan="2">ECTS: 7<br>divided into<br>ECTS lessons: 6.5<br>ECTS exe: 0.5</td></tr>
<tr><td colspan="6">Time management, hours, in–class study hours, out–of–class study hours<br>lesson: 52 exe: 8 in–class study: 60 out–of–class study: 115</td></tr>
<tr><td>Language:<br>Italian</td><td>Compulsory Attendance:<br>no</td><td colspan="4"></td></tr>
<tr><td>Subject Teacher:<br>Maria Falcitelli</td><td>Tel: 39 0805442844<br>e–mail:<br>maria.falcitelli@uniba.it</td><td>Office:<br>Department of Mathematics<br>Room 9, Floor 3</td><td colspan="3">Office days and hours:<br>by appointment</td></tr>
<tr><td colspan="6">Prerequisites: Mathematical knowledge acquired during the first degree in Mathematics. In particular: linear Algebra, general Topology, classical Mathematical Analysis, affine and projective Geometry, basic concepts occurring in differential Geometry</td></tr>
<tr><td colspan="6">Educational objectives: Acquiring new concepts and basic methods occurring in modern Differential Geometry, in particular in Riemannian Geometry.</td></tr>
<tr><td>Expected learning outcomes (according to Dublin Descriptors)</td><td colspan="5">Knowledge and understanding: Acquiring new concepts and methods of proof.<br>Applying knowledge and understanding: The acquired knowledge is useful in various contexts, such as in theoretical Physics.<br>Making judgements: Ability in recognizing new techniques used in problem solving.<br>Communication: Students should acquire the mathematical formalism which is necessary to analyze advanced problems.<br>Lifelong learning skills: Relating the main concepts occurring in various mathematical and Physical disciplines.</td></tr>
<tr><td colspan="6">Course program<br>Fundamental examples of smooth manifolds.<br>The Euclidean space $R^n$. The sphere $S^n(r)$. The real projective space $P_n(R)$ and the antipodal map. The hyperbolic space $H^n_r$.<br><br>The tensor algebra of a manifold.<br>The tensor algebra on a vector space. Tensor fields of type (r,s) on a manifold: definition and properties. The tensor algebra of a manifold. Contractions. Symmetric, skew-symmetric tensors on a vector space. Symmetric tensor fields, differential forms on a manifold. The exterior product and the algebra of differential forms. The exterior differential.<br><br>Derivations of the tensor algebra.<br>Definition and main properties of a derivation of the tensor algebra. Examples: the derivation associated with a (1,1)-tensor field, the Lie derivative with respect to a vector field. A representation theorem of derivations.<br><br>Linear connections.<br>Definition of a linear connection. The covariant derivative of a tensor field with respect to a connection. The canonical connection on $R^n$. The localizability property and a representation theorem. The covariant derivative of a vector field along a curve. Parallel vector fields, geodesic curves: definition and equations. The parallel transport along a curve. The torsion and the curvature tensors of a connection. Symmetric, flat connections. Bianchi identities.<br><br>Riemannian manifolds.</td></tr>
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Riemannian metrics on a manifold. The metric induced on a submanifold of a Riemannian manifold. Examples. The scalar product of two tensor fields. The musical isomorphisms. The gradient od a smooth function. The Levi-Civita connection on a Riemannian manifold and the Christoffel symbols. Examples. The parallel transport along a curve induced by the Levi-Civita connection. The distance between two points in a Riemannian manifold. Complete, geodesically complete manifolds. Conformal changes of a metric.

Riemannian curvature.
The Riemannian curvature tensor: definition and properties. Sectional curvatures. Manifolds with pointwise sectional curvature. The Schur lemma. Space-forms: definition and main examples. Riemannian covering spaces. Example: the n-sphere as a Riemannian covering of $P_n(R)$. Complete, connected, simply connected space-forms: a classification theorem. Ricci tensor and scalar curvature. Einstein manifolds. A characterization of Einstein manifolds in dimension 3.

Riemannian submanifolds.
Riemannian submanifolds of a Riemannian manifold: definition and examples. The normal bundle, normal vector fields. The Gauss and Weingarten equations. The second fundamental form, the Weingarten operators: definition and properties. The mean curvature vector. Totally geodesic, totally umbilical, minimal submanifolds. Principal curvatures. Some curvature properties of a submanifold: Gauss, Codazzi, Ricci equations. Hypersurfaces in $R^{n+1}$.

**Teaching methods:** Lectures and exercise lessons.

**Auxiliary teaching:**

**Assessment methods:**
Oral exam.

**Bibliography:**
M. Abate, F. Tovena: Geometria Differenziale, Springer
T. Aubin: A course in Differential Geometry, American Mathematical Society
B. Y. Chen: Geometry of submanifolds, Marcel Dekker
W. Klingenberg: Riemannian Geometry, Walter de Gruyter
S. Kobayashi, K. Nomizu: Foundations of Differential Geometry, Vol. I,II, Interscience Publishers
G. Walschap: Metric structures in Differential Geometry, Springer.