Academic subject: Geometry 3			
Degree Class: L-35	**Degree Course:** Mathematics	**Academic Year:** 2020/2021	
	Kind of class: Mandatory	**Year:** 2	**Period:** 1
		ECTS: divided into **ECTS lessons:** **ECTS exe/lab/tutor:**	

Time management, hours, in–class study hours, out–of–class study hours
lesson: 40 exe/lab/tutor: 30 in–class study: out–of–class study:

Language:	**Compulsory Attendance:**		
Italian	no		
Subject Teacher: Antonio Lotta	**Tel:** 080 5442656 **e–mail:** antonio.lotta@uniba.it	**Office:** Department of Mathematics Room 7, Floor 2	**Office days and hours:** By appointment

Prerequisites: Mathematical knowledge which is usually acquired during the first year of the degree in Mathematics. Especially: linear algebra, affine and Euclidean spaces.

Educational objectives: Acquiring the basic concepts in Projective Geometry and in the theory of conics and quadrics.

Expected learning outcomes (according to Dublin Descriptors)

Knowledge and understanding: Acquiring fundamental concepts and classical geometrical methods using a modern language.

Applying knowledge and understanding: The knowledge acquired has a wide spectrum of applications, both in the field of pure mathematics and other scientific disciplines, for example in computer science (3D graphics, design, robotics, computer vision, etc.)

Making judgements: Ability in developing new methods which are useful in problem solving.

Communication: Students should acquire the Mathematical language and formalism which are necessary to analyze and solve problems.

Lifelong learning skills: Acquiring suitable learning methods and relating the main concepts occurring in various mathematical disciplines.

Course program

Projective spaces. The projective space P(V) associated to a vector space V. Projective spaces KP^n. Projectively independent points. Subspaces; lines, planes, hyperplanes. Intersection of subspaces and projective subspace joining a finite family of subspaces. Projective Grassmann formula. Cartesian and parametric equations of a subspace. Linear system of hyperplanes containing a given subspace. Projective transformations of KP^n, projective transformation group. Points in general position and relative characterization. Existence and uniqueness theorem for projective transformations. Models of projective spaces over a field K. Coordinated systems. Field expanded with the element at infinity. and its canonical structure as a model of projective line. Projective completion S(A) of an affine space A with the addition of directions. Coordinated systems admissible on S(A) deduced from affine frames. Projective transformations between projective spaces: characterization, properties, equations, existence and uniqueness theorem. Projective transformations transform subspaces into subspaces. Structure of projective space induced on a set by transport by means of a bijection. Canonical structure of projective space induced on a subspace. Dual of a projective space. Linear systems of hyperplanes considered as projective subspaces of the dual space. Examples: pencils of lines and pencils of planes. Projection from a point to a pencil of lines in a projective plane. Characterization of the projective subspaces of S(A) which are not contained in the hyperplane at infinity as extensions S(E) of affine subspaces E of A. Link between the equations of an affine subspace and those of its projective completion. Canonical extension of an affinity to a projective transformation. Canonical isomorphism between the group of affinities and the group of projective transformations that preserve the hyperplane at infinity.

Projective geometry in one dimension. Non-homogeneous projective coordinate (projective abscissa); relevant special cases. The bilinear equation of a projective transformation between projective lines. Cross ratio. Harmonic conjugate point of an ordered triple of points. Characterization of the projective transformations between projective lines as bijective maps preserving the cross ratio. Projective transformations that transform an ordered set of four distinct points into another. Elliptic, parabolic, hyperbolic projectivities. Involutions. Fixed points of a projectivity. The characteristic of a hyperbolic projectivity. Existence and uniqueness of the hyperbolic projectivity of assigned fixed points joined and assigned characteristic. The circular involution on a proper pencil of lines in the projective completion of a Euclidean plane.

Projective hyperquadrics. Quadratic forms and symmetric bilinear forms: signature, radical. Hyperquadrics of a projective geometric space of dimension $n \geq 1$. Support of a hyperquadric. The set of hyperquadrics is a projective space of dimension $n(n + 3)/2$. Rank of a hyperquadric. Image of a hyperquadric through a projective transformation.
Projective classification of hyperquardircs in the complex case. Maximum dimension of a subspace contained in a complex non-degenerate hyperquadric (statement only). Index of a hyperquadric of a real projective space and its geometric meaning. Classification theorem in the real case. The case of conics and quadrics. Definition of elliptic quadric and hyperbolic quadric. Intersection between a hyperquadric and a projective subspace; relative positions between a line and a hyperquadric. Singular points of a hyperquadric, radical. Characterization of the hyperquadrics which coincide with the radical. Structure of degenerate hyperquadrics that do not coincide with their radical: projective cones. Examples in dimension two and three. Polarity. Conjugated points. Polar hyperplane of a non-singular point. The polarity defined by a non-degenerate hyperquadric is a projective transformation. Discussion of the intersection between a hyperquadric and the polar hyperplane of a point. Tangent hyperplanes. Tangents conducted from a point to a conic. Elliptic points and hyperbolic points of a non-degenerate real quadric. The involution of the conjugate points on a secant or external line to a hyperquadric. Lines which are conjugated with respect to a conic, involution of the conjugated lines passing from a fixed a point.
Affine properties of hyperquadrics. The center of a non-degenerate hyperquadric of the projective completion of an affine space. Central hyperquadrics. Definition of ellipse, hyperbola, parabola and their characterizations. Diameters of a conic, conjugated diameters. Asymptotes of a hyperbola. Conjugation of diameters. Definition of ellipsoid, hyperboloid and paraboloid. Notion of affine hyperquadric and its projective closure. The bijective correspondence between affine hyperquadrics and projective hyperquadrics that do not contain the hyperplane at infinity. Affine equivalence criterion for hyperquadrics (statement only). Canonical equations of affine hyperquadrics. The case of conics and quadrics. Cones and cylinders. Practical methods to classify a real conic or a real quadric from the projective or affine point of view. The notion of pencil of hyperquadrics. Description of the various types of conic pencils.
Metric properties of hyperquadrics. Hyperspheres viewed as special hyperquadrics of a Euclidean space. Principal hyperplanes of a non-degenerate hyperquadric. Axes of a conic and its vertices. Standard equation of a conic. Round quadrics. The foci of a conic. Eccentricity.

Teaching methods: Lectures and exercise sessions.

Auxiliary teaching: Auxiliary didactic material available on the teacher's homepage (https://www.dm.uniba.it/members/lotta)

Assessment methods: Written and oral Exam

Bibliography:
M. Beltrametti, E. Carletti, D. Gallarati, G. Monti Bragadin: *Lezioni di Geometria analitica e proiettiva*, Bollati Boringhieri, 2003.
M. Berger: *Geometry II*, Universitext, Springer-Verlag, 1987.
E. Casas-Alvero: *Analytic Projective Geometry,* EMS Textbooks in Mathematics. European Mathematical Society (EMS), 2014.
E. Sernesi: *Geometria 1,* Bollati Boringhieri, 1994.
E. Fortuna, R. Frigerio, R. Pardini: *Geometria proiettiva, problemi risolti e richiami di teoria*, Springer-Verlag, Collana Unitext, 2011.