Academic subject: Geometry 2

Degree Class:	**Degree Course:**	**Academic Year:**	
L-35 - Scienze Matematiche	Mathematics	2020/2021	
	Kind of class: Mandatory	**Year:** 1	**Period:** II semester
		ECTS: 8 divided into **ECTS lessons:** 5 **ECTS exe/lab/tutor:** 3	

Time management, hours, in–class study hours, out–of–class study hours
lesson: 40 exe/lab/tutor: 30 in–class study: 70 out–of–class study: 130

Language:	**Compulsory Attendance:**		
Italian	no		
Subject Teacher:	**Tel: 080 5442679** **080 5442647** **e–mail:**	**Office:** Department of Mathematics	**Office days and hours:** see webpages
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Prerequisites: Basic knowledge of linear algebra: vector spaces, linear maps and bilinear forms.

Educational objectives: Acquiring language and techniques of affine and Euclidean geometry.

Expected learning outcomes (according to Dublin Descriptors)

Knowledge and understanding:
Acquiring fundamental concepts in affine and Euclidean geometry. Acquiring basic mathematical proof techniques.

Applying knowledge and understanding:
Students should become able to use the acquired theoretical knowledge in solving problems.

Making judgements:
Ability to analyze the consistency of the logical arguments used in a proof.

Communication:
Students should learn the mathematical language and formalism necessary to explain the acquired knowledge and solve problems.

Lifelong learning skills:
Acquiring suitable learning methods necessary to read and understand texbooks dealing with the program topics.

Course program

Euclidean vector spaces.
Scalar products on real vector spaces. The norm of a vector. Orthogonal and orthonormal vectors. Orthogonal complement of a vector subspace. Orthonormal bases. Gram-Schmidt process. Angle between two vectors. Selfadjoint endomorphisms. Spectral Theorem. Unitary operators. Orthogonal matrices. Rotations and reflections.

Affine spaces.
Geometric vectors. Affine spaces associated to vector spaces: definition, elementary properties and first examples. Vector spaces as affine spaces. Affine frames and coordinate systems. Change of affine frames. Orientation of a real affine space. Barycenter of weighted points.

Affine subspaces.
Affine subspaces and their direction. Affine space structure induced on an affine subspace. Characterization of affine subspaces via barycenters. Affine subspace spanned by a finite number of points: definition and characterizations. Affinely independent points. Collinear and coplanar points. Ratio of three collinear points.

Intersection of affine subspaces. Parallel subspaces. The affine subspace spanned by two subspaces. Affine Grassmann identity. Coplanar lines. Parametric and cartesian equations of an affine subspace.

Affine geometry in dimension 2. Parametric equations and cartesian equation of a line. Coordinate axes. Parallel lines and intersection of lines. Sheaves of lines.

Affine geometry in dimension 3. Parametric and cartesian equations of a plane. Parallel planes. Intersection of planes. Parametric and cartesian equations of a line. Coordinate axes and planes. Parallel lines. Parallelism between a line and a plane. Coplanar lines. Sheaves of planes.

Euclidean spaces.
Euclidean space associated to a Euclidean vector space. Cartesian frames and cartesian coordinates. Change of cartesian frames. Distance between two points. Euclidean affine subspaces. Angles between two lines. Convex angle between two oriented lines. Orthogonal lines. Orthogonal subspaces. Orthogonal projection of a point onto a subspace. Distance of a point to a subspace. Euclidean line.

Euclidean geometry in dimension 2. Angles between lines. Orthogonal lines. Angles between a line and coordinate axes. Angular coefficient of a line. Distance of a point to a line. Circles.

Euclidean geometry in dimension 3. Angles between lines. Orthogonal lines. Angles between a line and coordinate axes. Angles between planes and orthogonal planes. Angle between a line and a plane. Orthogonality between a line and a plane. Distance of a point to a plane. Distance of a point to a line. Minimum distance between lines. Spheres and circles. Surfaces of revolution: cones and cylinders.

Affine maps and affine transformations.
Affine maps: definition and first properties. Characterization of affine maps via barycenters. Example: projection parallel to a direction onto an affine subspace. Images of affine subspaces under affine maps. Affine transformations. The affine group. Existence and uniqueness of affine transformations. Equations of an affine transformation. Affinely equivalent sets. Translations: definition, characterization and equations. Fixed points of an affine transformation. Affine transformations with a fixed point. Decomposition of an affine transformation. Homotheties and symmetries.

Isometries.
Isometries of a Euclidean space: definition and characterization. Isometries and angles between lines. Existence and uniqueness of isometries. Isometric sets. Equations of an isometry. Direct isometries and opposite isometries. Rotations and reflections. Decomposition of isometries. Reflections with respect to a hyperplane. Decomposition of isometries into reflections with respect to hyperplanes. Isometries of the Euclidean plane: translations, rotations, reflections, glide reflections. Chasles' Theorem. Rotations in the 3-dimensional Euclidean space. Hints on the classification of isometries in the 3-dimensional Euclidean space.

Affine and Euclidean conics.
Conics in the real or complex affine plane. Equation of a conic. Affine and Euclidean classification of conics, and their canonical forms.

Teaching methods:
Lectures and exercise sessions.

Auxiliary teaching:
Textbooks in bibliography.
Notes and exercise files available at the teachers' webpages or on Microsoft Teams platform.
Tutoring activity (https://www.dm.uniba.it/didattica/tutorato)

Assessment methods:
Written and oral exam. Joint exam with Geometry 1.

Bibliography:

E. Abbena, A.M. Fino, G.M. Gianella, *Algebra lineare e geometria analitica*, Aracne.
S. Abeasis, *Algebra lineare e Geometria*, Zanichelli.
M. Audin, *Geometry*, Universitext, Springer.
M. Berger, *Geometry I*, Universitext, Springer.
G. Campanella, *Affinità, isometrie, proiettività*, Aracne.
E. Sernesi, *Geometria 1*, Bollati Boringhieri.
M.I. Stoka, *Corso di Geometria*, Ed. Cedam Padova.