Academic subject: **Numerical Analysis**				
Degree Class: LM-40 – Matematica		**Degree Course:** Mathematics	**Academic Year:** 2020/2021	
		Kind of class: Mandatory/optional	**Year:**	**Period:**
			ECTS: 7 divided into **ECTS lessons: 6.5** **ECTS exe/lab/tutor: 0.5**	
Time management, hours, in–class study hours, out–of–class study hours lesson: 52 exe/lab/tutor: 8 in–class study: 60 out–of–class study:115				
Language: Italian	**Compulsory Attendance:** no			
Subject Teacher:	**Phone and e–mail:**	**Office:**	**Office days and hours:**	
Roberto Garrappa	080.5442685 roberto.garrappa@uniba.it	Dept. of Mathematics Room 7, Floor 3	Wednesday 15:00-17:00 (Other days by appointment)	
Francesca Mazzia	0805442702 francesca.mazzia@uniba.it	Dept. of Mathematics Room 7, Floor 4	Wednesday 11:30-13:30 (Other days by appointment)	
Prerequisites: Mathematical knowledge which usually is acquired during the years of a degree of L–35 class. Especially: Numerical calculus, linear algebra and programming.				
Educational objectives: Acquiring knowledge of numerical methods for the solution of differential equations and large linear systems.				

Expected learning outcomes (according to Dublin Descriptors)

Knowledge and understanding:

- Learn the techniques for the numerical programming of numerical methods for the solution of differential equations and large linear systems by means of iterative methods.

Applying knowledge and understanding:

- Acquiring the ability to solve differential equations using optimized algorithms with good stability problems.
- Acquiring the ability to programming, testing interpreting the results correctly.
- Acquiring the ability to solve mathematical problems using problem solving environment.

Making judgements: acquiring ability to find the most suitable numerical method for the solution of a differential problem.

Communication: acquiring ability to rigorously define the mathematical problem studied in the course and to expose its numerical methods, outlining its fundamental properties

Lifelong learning skills: ability to study and solve problems similar but not necessarily the same as those dealt with during lessons.

Course program

1. Numerical solution of differential equations, initial value problems: linear multi-step methods, Adams methods, BDF methods, MEBDF methods; Consistency, convergence and 0-stability; root conditions. Absolute and relative stability; A-stability; stiff problems, error estimation and step-variation strategies. Solution of test problems in R and/or Matlab.

2. Numerical solution of boundary value differential equations: Dicotomy and conditioning, finite difference schemes for first order and second order problems, collocation methods, mono implicit Runge-Kutta methods, boundary value linear multistep method, deferred correction, extrapolation techniques, error estimation and mesh selection. Solution of test problems in R and/or Matlab.

3. Numerical solution of partial differential equations: advection-diffusion equations (heat equation, advection equation, Laplace equation), finite difference methods, CFL condition. Semidiscretization methods, the method of lines. Staggered mesh and finite volume methods. Boundary conditions, Crank-Nicholson method, Stability and convergence for the semidiscretized problem e for the total discretization. Note on Fourier analysis and eigenvalue analysis. Variational formulation and finite element method for one dimensional problems. Solution of test problems in R and/or Matlab.

3. Numerical solution of partial differential equations. Poisson and Laplace equations. Finite differences methods: 5-points and 9-points stencils. Ordering of variables. Dirichlet and Neumann boundary conditions. Consistency and convergence, inverse of discretization matrix bounded in norm, ill-conditioning. Evolutionary problems. Diffusion equation: explicit schemes and stability issues; the method of lines; consistency, stability and convergence. Crank-Nicolson method. Advection equations: generality and theoretical solution; stability issues with forward differences. Mid-point and Leapfrog method; Lax-Friederisch method; boundary numerical conditions. Fourier and eigenvalues analysis. Variational formula and finite element methods. Matlab programming.

4 Numerical methods for the solution of large systems of algebraic equations. Splitting methods and convergence for problems coming from discretization of Poisson equations. Krylov subspace methods: theory and implementation. Arnoldi algorithm and Lanczos symmetric algorithm; FOM , MinRes, GMRes and GC methods. Restart. Convergence.

Teaching methods:
Lectures and exercises on the implementation of numerical schemes.

Auxiliary teaching:

The suggested books can be completed by slides and other possible didactic material from the teacher.

Assessment methods: Oral exam.

Bibliography:

R. LeVeque, Finite Difference Methods for Ordinary and Partial Differential Equations: Steady State and Time Dependent Problems. SIAM, 2007

Y. Saad, Iterative Methods for Sparse Linear Systems, SIAM, 2013

U.M. Asher, Numerical methods for evolutionary differential equations, SIAM 2008

K. Soetaert, J. Cash, Jeff, F. Mazzia, Solving Differential Equations in R, Springer, 2012

U.M. Ascher, R.M. Mattheij and R.D. Russell, Numerical Solution of Boundary Value Problems for Ordinary Differential Equations, SIAM 1995,