<table>
<tr><td colspan="5">Academic subject: Functional Analysis</td></tr>
<tr><td colspan="2">Degree Class:
L-35- Scienze Matematiche</td><td colspan="2">Degree Course:
Mathematics</td><td>Academic Year:
2020/2021</td></tr>
<tr><td colspan="2"></td><td colspan="2">Kind of class:
optional</td><td>Year: | Period: 2</td></tr>
<tr><td></td><td></td><td colspan="2"></td><td>ECTS: 7
divided into
ECTS lessons: 6,5
ECTS exe: 0,5</td></tr>
<tr><td colspan="5">Time management, hours, in–class study hours, out–of–class study hours
lesson: 52 exe: 8 in–class study: 60 out–of–class study: 115</td></tr>
<tr><td>Language:
Italian</td><td>Compulsory Attendance:
no</td><td colspan="3"></td></tr>
<tr><td>Subject Teacher:
Genni Fragnelli</td><td>Tel: +39 080 544 2687
e-mail:
genni.fragnelli@uniba.it</td><td>Office:
Department of Mathematics
Room 10, Floor III</td><td colspan="2">Office days and hours:
By appointment via email</td></tr>
<tr><td colspan="5">Prerequisites:
Mathematical knowledge which usually is acquired during the first two years of a degree of L-35 class. Especially: classical analysis of one and several variables, normed spaces, general topology, linear algebra.</td></tr>
<tr><td colspan="5">Educational objectives:
Acquiring language and basic tools concerning functional spaces, representation theorems, operator theory and operator semigroups, with applications to some classes of partial differential equations</td></tr>
<tr><td>Expected learning outcomes (according to Dublin Descriptors)</td><td colspan="4">Knowledge and understanding:
Acquiring fundamental concepts and results in the setting of functional spaces and operator theory. Acquiring main tools and proof techniques

Applying knowledge and understanding:
The acquired theoretical knowledge find many applications in several aspects of mathematics, including partial differential equations and related models.

Making judgements:
Ability to analyze the consistency of the logical arguments used in a proof, problem solving skills and ability to choose suitable mathematical tools consistent with the theoretical knowledge.

Communication:
Acquiring mathematical language and formalism necessary to read and understand textbooks, to explain the acquired knowledge, and to describe, analyze and solve problems.

Lifelong learning skills:
Acquiring suitable learning methods, supported by consultation of texts and by solving exercises and problems related to the contents of the course.</td></tr>
</table>

Course program

1. **Normed spaces and fundamental theorems**

Normed spaces. Normed algebras. Linear operators and bounded linear operators on normed spaces. Banach spaces. Dual space of a normed space. Weak topology. Relations between a normed space and its dual space. Reflexive spaces. Finite dimensional normed spaces. Riesz Theorem on the dimension. Baire spaces. Uniform boundedness Theorem. The Banach-Steinhaus Theorem. The Open Mapping Theorem and applications. The Closed Graph Theorem and applications. The Hahn-Banach Theorem and applications. Hilbert spaces. Orthogonal projections. The Riesz representation theorem. Sesquilinear forms. The Lax-Milgram Theorem.

2. **Linear operator on normed spaces**

Bounded linear operators on normed spaces and their adjoints. Finite-rank operators, approximable operators, compact operators. Fredholm operators and their index. A fundamental theorem on compact operators. Fredholm alternative. Basic spectral theory for linear operators. The spectrum of a compact operator. The Neumann representation theorem. Existence of spectral values. Closed linear operators. Graph norm. Closable operators and their closures. Spectral properties of closed linear operators. Resolvent operator and its properties.

3. Linear operators on Hilbert spaces

Bounded linear operators on Hilbert spaces and their adjoints. Skew-adjoint and self-adjoint operators. Representation theorems. Positive operators. Normal operators. Spectral representation theorems. Closed linear operators and their adjoints.

4. Operator semigroups on Banach spaces

Strongly continuous semigroups, groups of operators on a Banach space. Significant examples. Exponential inequality and growth bound. The generator of a strongly continuous semigroup and its properties. Significant examples. Abstract Cauchy problems and semigroups. The Hille-Yosida Theorem. Dissipative and m-dissipative operators. The Lumer-Phillips Theorem. The heat equation in $L^2(0,1)$. Perturbations of generators. Regularity of operator semigroups.

Teaching methods:
Lectures and exercise sessions

Auxiliary teaching:

Assessment methods:
Oral exams

Bibliography:

[B1] H. BREZIS, Analyse fonctionnelle, Theorie et applications, 2e tirage, Masson 1987.

[B2] H. BREZIS, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer, 2011.

[EN] K.J. ENGEL - R. NAGEL, One-parameter Semigroups for Linear Evolution Equations, Graduate Texts in Mathematics 194, Springer, 2000.

[G] J.A. GOLDSTEIN, Semigroups of Operators and Applications, Second Edition, Dover Publications, Inc. New York 2017.

[L] P.D. LAX, Functional Analysis, Wiley Interscience, 2002.

For the topics in 1.-3. we will refer to [B1], [B2] and [L]. For the topics in 4. we will refer to [EN] and [G].