Academic subject: Fourier Analysis and Potential theory				
Degree Class: LM–40 – Matematica		**Degree Course:** Mathematics	**Academic Year:** 2020/2021	
		Kind of class: optional	**Year:** 2	**Period:** 1
			ECTS: 7 divided into **ECTS lessons:** 6 **ECTS exe/lab/tutor:** 1	
Time management, hours, in–class study hours, out–of–class study hours lesson: 48 exe/lab/tutor: 12 in–class study: 60 out–of–class study: 115				
Language: Italian and English	**Compulsory Attendance:** no			
Subject Teacher: Marcello D'Abbicco Cooperates: Annunziata Loiudice	**Tel:** 080 544 2721 **e–mail:** marcello.dabbicco@uniba.it	**Office:** Department of Mathematics Room 36, Floor 2	**Office days and hours:** by appointment via e-mail	
Prerequisites: Mathematical knowledge which usually is acquired during a degree of L–35 class. Especially: classical analysis of one and several variables, general topology, linear algebra, Lebesgue measure and integration theory.				
Educational objectives: Acquiring language and techniques of modern analysis, especially interpolation theorems, maximal functions, L^p-bounded operators, Riesz potential theory, singular integral operators, multipliers theorems, application to linear and semilinear evolution equations; homogeneous Lie groups and sublaplacians, Heisenberg group and the Kohn Laplacian, Haar measures.				
Expected learning outcomes (according to Dublin Descriptors)	**Knowledge and understanding:** Acquiring fundamental concepts in advanced modern real analysis and Fourier Analysis. Acquiring mathematical proof techniques and learn the applications to linear and semilinear partial differential equations. **Applying knowledge and understanding:** The acquired theoretical knowledge is useful in large part of mathematics and its applications. **Making judgements:** Ability to analyze the consistency of the logical arguments used in a proof. Problem solving skills should be supported by the capacity in evaluating the consistency of the found solution with the theoretical knowledge. **Communication:** Students should acquire the mathematical language and formalism that are necessary to read and comprehend textbooks, to expound the acquired knowledge, and to describe, analyse and solve problems. **Lifelong learning skills:** Acquiring suitable learning methods, supported by text consultation and by solving the exercises and questions periodically suggested during the course.			
Course program 1. Fourier Analysis Basics on L^p spaces, convolution product, approximation of identity, Fourier transform, Schwartz space and tempered distribution space. Weak (p,q) convergence. Marcinkiewicz interpolation theorem. Hardy-Littlewood maximal function. Diadic maximal function. Calderón-Zygmund decomposition. Poisson kernels, P.V. 1/x, Hilbert transform. Riesz-Kolmogorov theorem. Multipliers. Singular integral operators. Fourier transform of P.V. $\Omega(x)/\lvert x\rvert^n$. Method of rotations. Riesz transforms. Riesz and Bessel potentials, fractional Soboleve spaces. Hardy-Littlewood-Sobolev theorem and Sobolev embedding theorems. Calderón-Zygmund theorem. Pseudo-differential operators. Real Hardy spaces H^p, atomic decomposition, BMO, operators in Hardy spaces and in BMO. Weighted inequalities with A_1 and A_p weights. Paley-Littlewood decomposition. Mikhlin-Hörmander multiplier theorems. Results for parameter-dependent multiplier theorems and application to evolution equations.				

2. Analysis on homogeneous Lie groups
Homogeneous Lie groups and sublaplacians. Heisenberg group and the Kohn Laplacian. Homogeneous norms. Haar measures. Convolution on groups. L^p-weak spaces and functional inequalities. Fundamental solution for sublaplacians. Representation formulas. Maximum principle. Hardy-Littlewood-Sobolev theorem for sublaplacians.

Teaching methods:
Lectures and exercise sessions.

Auxiliary teaching:
Didactic material available at www.dabbicco.com

Assessment methods:
Oral exam or research talk.

Bibliography:

J. Duoandikoetxea, Fourier Analysis, Graduate Studies in Mathematics, Vol 29, AMS, 2000.
M.R. Ebert, M. Reissig, Methods for Partial Differential Equations, Birkhäuser Basel, 2018.
L. Grafakos, Classical Fourier analysis. Third edition. Graduate Texts in Mathematics, 249. Springer, New York, 2014

Other didactic material (see above).