Academic subject: ALGEBRA 1

Degree Class: L-35 - Scienze matematiche	**Degree Course:** Mathematics	**Academic Year:** 2020/2021	
	Kind of class: mandatory	**Year:** 2	**Period:** 1
		ECTS: 8 divided into **ECTS lessons: 5** **ECTS exe/lab/tutor: 3**	

Time management, hours, in–class study hours, out–of–class study hours
lesson: 40 exe/lab/tutor: 30 in–class study: 70 out–of–class study: 100

Language: Italian	**Compulsory Attendance:** no		
Subject Teacher: Margherita Barile	**Tel:** +39 080 5442204 **e–mail:** margherita.barile@uniba.it	**Office:** Department of Mathematics 2nd floor, room 23	**Office days and hours:** By appointment

Prerequisites:
Basic concepts of set theory, including maps, relations, number sets.

Educational objectives:
Acquiring a solid knowledge of algebraic structures.

Expected learning outcomes (according to Dublin Descriptors)	**Knowledge and understanding:** Recognizing algebraic structures and their properties. **Applying knowledge and understanding:** Solving algebraic problems by means of a structural approach. **Making judgements:** Assessing the correctness of numerical results by reference to a conceptual framework. **Communication:** Formulating definitions and abstract arguments in a formally rigorous manner. **Lifelong learning skills:** Establishing logical connections between different topics.

Course program

Number sets:
The divisibility relation in $\mathbb{Z}$, prime numbers. Euclidean division in $\mathbb{Z}$. The GCD and Bézout's Identity, the Euclidean algorithm. The Fundamental Theorem of Arithmetic. Euclid's theorem on the infinitude of primes. The complex numbers as ordered pairs of real numbers, operations in $\mathbb{C}$, algebraic and trigonometric form, the *n*-th roots of a complex number, the Fundamental Theorem of Algebra.

Algebraic structures and homomorphisms:
Elementary properties of groups, Abelian groups, subgroups, cyclic groups and their generators, order of a periodic element, Lagrange's Theorem on the order of elements in a finite Abelian group. Elementary properties of rings, commutative rings, unit rings, invertible element, integral domains, division rings, fields, subrings, subfields. Rings of matrices. Direct product of groups and rings. Homomorphisms, monomorphisms, epimorphisms, isomorphisms, kernel of a homomorphism.

Polynomial rings:
Polynomials in one indeterminate, degree of a polynomial. Operations on polynomials. Polynomials with coefficients in an integral domain, the degree formula. Euclidean division in $K[x]$. The GCD and Bézout's Identity, the Euclidean division algorithm. Roots of a polynomial, Rational root theorem. Irreducible polynomials, factorization, associate polynomials. Algebraically closed fields. Gauss' Theorem and factorizations in $\mathbb{Q}[x]$. Reduction modulo *p*, Eisenstein's irreducibility criterion. Irreducible polynomials in $\mathbb{C}[x]$ and in $\mathbb{R}[x]$.

Quotient structures:
The congruence modulo n in $\mathbb{Z}$. The residue class ring $\mathbb{Z}_n$. Linear congruences in $\mathbb{Z}$ and linear equations in $\mathbb{Z}_n$. The group of units of $\mathbb{Z}_n$. The fields $\mathbb{Z}_p$. The Euler function. The Chinese Remainder Theorem. Fermat's little Theorem, the Euler Theorem. The congruence modulo $f(x)$ in $K[x]$. The residue class ring $K[x]/f(x)$ and its units.

Symmetric groups:
The natural action of S_n on $X = \{1,...,n\}$. Orbits and cycles of a permutation. Decomposition of a permutation into disjoint cycles. Parity of a permutation, the alternating group A_n.

Teaching methods:
(Online) lectures and exercise sessions.

Auxiliary teaching:
Material available on line:

- Complete lecture notes
- Exam sheets
- Collections of exercises
- Additional course material (historical notes, solved exercises, further remarks) uploaded onto the Microsoft Teams platform

http://www.dm.uniba.it/~barile/Rete/indice.htm

Assessment methods:
Written exam and oral exam.

Bibliography:

Appunti di Algebra 1, Giulio Campanella (Nuova Cultura)
Algebra, G.M. Piacentini Cattaneo (Decibel- Zanichelli)
Elementi di Algebra, S. Franciosi, F. de Giovanni (Aracne Editrice)
Algebra, I.N. Herstein (Editori Riuniti)
Aritmetica e algebra, D. Dikranjan, M.S. Lucido (Liguori Editore)