Academic subject: Critical Points Theory			
Degree Class: LM – 40 Matematica		**Degree Course:** Mathematics	**Academic Year:** 2018/2019
		Kind of class: Optional	**Year:** / **Period:** 2
			ECTS: 7 divided into **ECTS lessons:** 6,5 **ECTS exe/lab/tutor:** 0,5
Time management, hours, in–class study hours, out–of–class study hours lesson: 52 exe/lab/tutor: 8 in–class study: 60 out–of–class study: 115			
Language: Italian	**Compulsory Attendance:** no		
Subject Teacher: Addolorata Salvatore	**Tel:** +39 0805442705 **e–mail:** addolorata.salvatore@uniba.it	**Office:** Department of Mathematics Room 10 , Floor IV	**Office days and hours:** Tuesday 11-13 Other days and times by appointment

Prerequisites:
Mathematical knowledge which usually is acquired during the first three years of a degree of L-35 class. Especially: classical analysis of one and several variables, general topology, linear algebra, Hilbert spaces and L^p spaces.

Educational objectives:
Acquiring knowledge of variational and topological methods in the study of nonlinear problems, with particular reference to topological degrees and index theories. Applications for the study of some semilinear elliptical problems.

Expected learning outcomes (according to Dublin Descriptors)

Knowledge and understanding:
Acquiring concepts and advanced techniques in the study of variational problems.

Applying knowledge and understanding:
The acquired theoretical knowledge is used in the study of different nonlinear differential problems.

Making judgements:
Ability to apply the mathematical tools available to study nonlinear problems coming from applied sciences.

Communication:
Students should acquire the mathematical language and formalism necessary to read and comprehend textbooks, to explain the acquired knowledge and to describe , analyze and solve problems.

Lifelong learning skills:
Acquiring suitable learning methods, supported also by consultation of the texts and by solution of problems suggested during the course.

Course program

Topological degree and applications: Topological degree for continuous functions in finite dimension: axiomatic definition and properties. Construction of the topological degree. Brower fixed point Theorem. Retraction Theorem. Borsuk Theorem. Topological linking, definition and examples: mountain pass linking, multidimensional mountain pass linking, saddle linking. Topological degree in infinite dimension. Schauder fixed point Theorem. Retraction Theorem. Other fixed point Theorems.

Index theory: Index theory in topological spaces: axiomatic definition. Lusternik-Schnirelmann category: definition, examples and properties. Krasnoselski genus: definition, examples and properties. Relationship between the degree and the category of a set. Index theory related to a group of unitary transformations on a Hilbert space. S^1- index.

Abstract theorems of critical points existence and applications: Generalities on the Palais-Smale condition.

Deformation Lemma. Linking Theorem and applications to the study of some elliptic problems with sublinear or superlinear growth. Linking Theorem for strongly indefinite functionals. Application to the study of a first order hamiltonian system.

Abstract theorems of critical points multiplicity and applications
Deformation lemma for functionals compatible with an index theory. Abstract theorems of critical points multiplicity for functionals bounded from below and compatible with an index theory. Abstract theorems using the Lusternik-Schnirelmann category and applications. Study of a nonlinear eigenvalue problem. Abstract theorems of critical points multiplicity for functionals even and bounded from below. Applications to some symmetric elliptic equations. Symmetric mountain pass theorem and symmetric multidimensional mountain pass theorem. Pseudo-index theory. An abstract theorem of critical points multiplicity for functionals even and unbounded from below. Applications to some superlinear or asymptotically linear elliptic problems. Multiplicity results for strongly indefinite S^1 - invariant functionals. Application to the study of symmetric first order hamiltonian system.

Teaching methods:
Lectures and exercise sessions.

Auxiliary teaching:

Assessment methods:
Oral exam.

Bibliography:

P.H. Rabinowitz, Minimax methods in critical point theory with applications to differential equations, CBMS Regional conference Series in Applied Mathematics, 65 (1986).

J.T. Scharwtz, Nonlinear Functional Analysis, Gordon & Breach, New York (1969).

M. Struwe, Variational methods, 3rd edition, Springer- Berlin (2000).