

<b>Academic subject:</b> Numerical Methods and Modelling						
<b>Degree Class:</b> LM-40- Mathematics	<b>Degree Course:</b> Mathematics	<b>Academic Year:</b> 2018/2019				
	<b>Kind of class:</b> mandatory	<b>Year:</b> 1	<b>Period:</b> 1			
			<b>ECTS: 7</b> divided into <b>ECTS lessons:</b> 4 <b>ECTS exe/lab/tutor:</b> 3			
<b>Time management, hours, in-class study hours, out-of-class study hours</b> lesson: 40 exe/lab/tutor: 32 in-class study: 72 out-of-class study: 103						
<b>Language:</b> Italian	<b>Compulsory Attendance:</b> no					
<b>Subject Teacher:</b> Luciano Lopez	<b>Tel:</b> +39 080 5442655 <b>e-mail:</b> luciano.lopez@uniba.it	<b>Office:</b> Department of Mathematics Room 15 , Floor 2	<b>Office days and hours:</b> Wednesday 11-13			
<b>Prerequisites:</b> Basic knowledge about systems of differential equations, Linear Algebra and numerical analysis, of a programming language, that are acquired in the Bachelor of class L-35.						
<b>Educational objectives:</b> Acquiring methods and techniques of applied mathematics to the simulation of continuous and discrete models of evolution. Ability of mathematical modeling of simple phenomena.						
<b>Expected learning outcomes (according to Dublin Descriptors)</b>	<p><b>Knowledge and understanding:</b> Acquiring fundamental concepts of model analysis; equilibrium points, stability, limit cycles, long-term behaviors.</p> <p><b>Applying knowledge and understanding:</b> Acquiring methods for simulating discrete and continuous models and interpretation of results.</p> <p><b>Making judgements:</b> Ability to evaluate the correspondence of the models with the reality that they want to represent and possibly the ability to modify them.</p> <p><b>Communication:</b> Acquiring advanced mathematical language in the description of models and their simulation</p> <p><b>Lifelong learning skills:</b> Acquiring appropriate learning methods through systematic use of books, solution of exercises, and simulation of models by computers.</p>					
<b>Course program</b>						
<p><b>1. DISCRETE DYNAMICAL SYSTEM.</b></p> <p>First order difference equations and their solutions. Theory of linear difference equations of order k. Homogeneous difference equations. Computation of solutions. Characteristic polynomial: single roots and multiple roots. Operator <math>p(E)</math> and its properties. Computation of particular solutions. Equilibria of difference equations and their stability. Formal series. Variation of constants formula. Linear systems of difference equations. Stability of solutions of difference equations. Matrix functions and their properties. Asymptotic study of <math>A^n</math> and <math>\exp(tA)</math>. Discrete models: Cobweb simple and complex; Leslie model of population dynamics; Natchez Indian.</p> <p><b>2. CONTINUOUS DYNAMICAL SYSTEMS</b></p> <p>Autonomous linear systems of ODES: Principal matrix solution and its properties. Continuity of solutions with respect</p>						

to the initial condition and concept of stability. Equilibria and concepts of asymptotic stability, stability and instability; definitions and theorem. Invariant subspaces: stable, unstable and central space. Examples: planar linear systems; node, saddle point, focus and center. Autonomous non linear systems of ODES: properties of solutions. Differentiability with respect to the initial conditions. Equilibria and linearizations. Lyapunov functions. Periodic orbits and limit cycles. Long time behavior of solutions. Continuous models: Lotka-Volterra, competing species, Darwin.

### 3. RUNGE KUTTA METHODS

Consistency and convergence of Runge Kutta numerical methods. Local and global error, upper bounds. Verification of order of convergence in function of the discretization stepsize. Linear stability: stability function and region of absolute stability. Variable stepsize methods and Runge Kutta embedded methods. Performance of variable stepsize methods. Qualitative behavior of numerical methods: spurious fixed points.

**Teaching methods:**

Lectures and exercise sessions.

**Auxiliary teaching:**

Tutorial activity

**Assessment methods:**

Oral exam and computer simulation of models

**Bibliography:**

V. Lakshmikantham, D. Trigiante, Theory of difference equations: numerical methods and applications, Academic Press Inc, 1988.

D.G: Luemberger, Introduction to dynamic systems, J. Wiley and Sons, 1979.

M. Braun, Differential Equations and Their Applications: An Introduction to Applied Mathematics: An Introduction to Applied Mathematics. Springer, 1983.

J.D. Lambert, Numerical Methods for Ordinary Differential Systems: The Initial Value Problem, W